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THE

NEW PHYSICS.

A MANUAL OF EXPERIMENTAL STUDY FOR HIGH SCHOOLS AND PREPARATORY SCHOOLS FOR COLLEGE.

BY

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PREFACE.

THE experience of those who have made a careful study of the methods of instruction in physical science in the secondary schools, and who have judged of the results of these methods by the intellectual training manifested by students who present themselves for college, is unfavorable to the lecture or recitation system unsupported by laboratory work. While the student's knowledge of language in general has been acquired through many years of patient and long-continued work in what may be termed a literary laboratory, in which the implements or apparatus consist of grammars and dictionaries, his acquaintance with the subject of physics has often been gained by superficial study in a few weeks.

One of the most important factors in intellectual growth is the long and persistent exercise of the mind upon a subject sufficiently broad to afford this continuous effort. It is believed that the study of physical science, in a laboratory, affords an

opportunity for this strenuous intellectual effort. The literary habit of mind is acquired by long study of language; and the scientific habit or instinct does not require less cultivation than the literary instinct.

The teacher may grant the comparatively small result which is obtained from the study of physics by the method of lectures and recitations; but he will immediately ask, "How can we obtain the time for laboratory work in physics—crowded as we are with so many studies?" Moreover, the subject of physics is unlike that of chemistry or botany. In these subjects the cost of the apparatus and material is comparatively small. Each student can have his own tubes, his own reagents, and his own microscope. In physics it is very different. The apparatus is expensive. A spectroscope, for instance, would cost nearly as much as the entire apparatus in a chemical laboratory. These are strong objections, and need careful consideration. The time, however, which is now given to recitation from text-books could at least be devoted to laboratory work; and the lectures could be reduced in number and only given to direct the students in their laboratory work. More can be done with cheap apparatus than is generally supposed. It is one of the objects of my treatise to show this.

I have endeavored to put into the hands of the teacher a manual which will stand in the same re-

lation to physics that many of the excellent manuals in chemistry stand to the instruction of chemistry in the secondary schools. I have endeavored also to arrange the experiments so that there may be a logical connection between them. I have early in the treatise directed the mind of the student toward the doctrine of the conservation of energy, and to the observation of the truth that our knowledge is relative, and that there are not different kinds of forces. Thus, I have endeavored to give a clear conception of what may be termed the new physics, so that even the general reader, in glancing over the character and the mutual relation of the experiments, can see the tendency of physical science to-day. I have not hesitated to avail myself early of electrical methods to illustrate problems of motion, and I have treated the magnetic pendulum together with the gravitation pendulum.

It is thought by some that a student can not begin the study of mechanics, and physics in general, without a good knowledge of trigonometry. The writer believes that the necessary amount of geometry and trigonometry can be taught almost at one sitting. On page 58 will be found an exposition of the amount of trigonometry which is necessary in order to understand this treatise. It is related in the life of De Morgan, the mathematician, that his teachers found him dull in mathe-

matics. One day his parents saw him occupied with a pair of compasses, and when he was asked what he was doing, he replied that he was "drawing mathematics." It is not derogatory to the dignity of mathematics to approach the subject by means of a pair of compasses or a foot-rule or a metre-scale, instead of by the stately process of proposition-lemmas and corollaries ending by Q. E. D. If we examine our own progress in geometry we shall find that we accept many things as the result of the experience of measurement, and have forgotten the philosophical proofs. Thus we can by actual measurement ascertain the truth that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the base and altitude of the triangle, and, knowing this fact, we can make a great and rapid advance in our knowledge of the configurations about us. Yet if our advance must be checked until we can prove rigidly by geometry this proposition, the conceptions of the world of motion, which our minds long to seize, must wait upon a cast-iron method which is not always the best calculated for all minds. How many of us to-day—having for gotten the proof in Euclid of the Pythagorean proposition — can sit down and prove it without considerable effort? Yet we use it every day in extending our knowledge of physics. The same remark can be applied to our use of π, the ratio

of the circumference of a circle to its diameter. How many of us can go through the steps of the demonstration without effort? It may be said that we work to-day by means of the old discipline in rigid geometry. This is true to a certain extent. We also work in philology by means of our exact grammatical training; but the teachers in languages are everywhere insisting upon the advantages of reading at sight, accepting certain grammatical forms as true without carefully proving them at first; judging by experience—or what corresponds to our measurement by the compasses—of the truth of these forms.

I have treated trigonometry as a mere branch of our conception of motion on an inclined plane.

It may be urged that the strict geometrical proofs of the law of pulleys, of the law of the conical pendulum, of the law of the simple pendulum, are far more conclusive and exact than any experimental proofs, which must necessarily be imperfect, on account of friction of the apparatus and the difficulty of properly applying the forces. This is true; but the mind may easily grasp the geometrical proofs, and proceed with facility to the Q. E. D. of the proposition, yet may not have a full conception of the magnitude of the forces which are brought into play and their relative importance. Thus, one who understands the theory of centripetal force might be incompetent to esti-

mate the proper size for the fly-wheel of an engine. For he has not handled apparatus, and has not gained a certain balance of judgment which comes from actual knowledge of the relative magnitude of the forces in question. It is only by actually trying experiments in mechanics that a student realizes the importance of the units which must be adopted. One can readily follow a geometrical proof of the law of the screw, but at the same time fail to seize the point that all distances must be expressed in the same unit, and that the force and the weight must also be expressed in terms of a consistent unit. The cultivated scientific instinct, moreover, will often save one the expenditure of much time and labor on unprofitable experiments in many departments of human effort.

This practical knowledge, it may be said, should follow the theoretical knowledge. Thus the young lawyer gains first his theoretical knowledge of law, and obtains his balance of judgment in actual practice. The young medical student and the engineer should have laboratory practice, however, before they experiment upon human life. It is necessary for the student of science, whether he is to be a physicist, a chemist, a botanist, or a geologist, or whether he is to be a physician or an engineer, to obtain a certain balance of judgment, and to cultivate a certain scientific instinct, which comes from putting theories into practice. The mind, more-

over, must rest upon physical laws for a comparatively long period in order to understand their true significance. Five minutes might enable one to grasp the geometrical proofs of the laws of centripetal force, yet the reflection of days may be necessary to see their physical importance. A physical experiment is valuable as an intellectual exercise, compelling one to exercise judgment in the disposition of material, in the avoidance of errors, and leading one to study the influence of laws other than those which we are striving to prove. In the elementary study of physics, however, the proposition proved geometrically makes but a slight impression upon the mind unless its significance is seen from an industrious and patient application.

Many teachers also object to apparatus which will not give the most accurate results that are obtainable in the present state of science. The writer believes, however, from long experience, that a careful study of the sources of error of an imperfect piece of apparatus often brings to the attention of the student problems in mechanics which a nicely adjusted piece of apparatus would not suggest to him. Mathematical problems in physics have their place in teaching; but physics should not be made a means of teaching mathematics. I have, therefore, substituted experimental problems for the mathematical problems which are usually

given in treatises on natural philosophy, in the hope of cultivating the scientific instinct.

The natural progress of our study of any subject is from the qualitative, or the comparatively rough evidence of our senses, to the quantitative, or exact determination of how much there is of anything. I have given both qualitative and quantitative experiments, endeavoring to lead the way from the first rough trials to the more exact methods. I have also often given several methods of determining the same quantity, endeavoring to point out that any intelligent student can cultivate self-reliance by trying his experiment in different ways and, by his control methods, ascertain how near to the truth his results are without appealing to the authority of a teacher. This cultivation of a mental fiber is a valuable result of laboratory work in physics. This treatise can, therefore, be used by the teacher to illustrate what the subject of modern physics is by a course of lectures, in which the experiments are performed before the class. A certain number of the same experiments should afterward be performed by each student. Without much expense, each student can have his own specific-gravity bottle, his own spiral spring-balance, his own vernier gauge, his own battery, and his own galvanometer. The more expensive instruments can be assigned in rotation to different students.

The author recommends that from one to two lectures be given during the week. In these lectures the experiments should be performed which the students afterward perform themselves in the laboratory; the complete calculations should be given, and attention be called to the precautions to be taken. In this way the class is carried over the same ground at the same time, and less individual instruction has to be given by the instructor in the laboratory. This method requires a certain amount of duplication of apparatus. It is best that the class should be divided into sets of six or ten students, and a sufficient number of vernier gauges, of specific-gravity bottles, of lenses, of galvanometers, should be provided to enable the men to keep pace with each other. This, however, is not an essential point.

An elementary laboratory of physics for a school, with experiments properly selected—for it is not necessary to cover the whole ground of experimental physics in order to gain a large amount of intellectual discipline—need not cost more than a chemical laboratory such as is now provided in many high-schools. I have endeavored to describe only simple and inexpensive apparatus. The teacher can readily invent simple contrivances which in many cases will be better than those I have recommended. My endeavor has been to point out the way to a more rational method of studying physics.

In the appendix will be found additional directions for constructing simple apparatus.

It is not supposed that the student will perform all the experiments in this treatise during the time that can be devoted to physics in the secondary schools. The choice of experiments by the teacher must necessarily depend upon the apparatus on hand which can be modified for the purposes of this treatise, and upon the amount of appropriation which can be devoted to cheap apparatus.

It is best that the student should see a greater amount of work than he can accomplish by an easy performance. In the study of language it is not expected that the student in the preparatory schools shall be able to master all the difficulties in the grammar set before him. It is hoped that this treatise will show that the subject of physics also affords great opportunity for intellectual training, and can not be compassed in a cram of a few weeks. In the preparation of this manual I have found valuable suggestions in the treatises of Pickering, of Worthington, and also of Thresh; and I am also indebted to Mr. Harold Whiting, assistant in the Physical Laboratory of Harvard University, for interesting modifications of certain experiments.

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CHAPTER I.

GENERAL IDEAS OF LENGTH AND VOLUME.

OUR knowledge of the world about us depends upon measurements. These measurements can be of two kinds—qualitative and quantitative. The child generally makes qualitative measurements in order to ascertain the nature and extent of its surroundings. Thus, one learns that one body is heavier than another, that the distance between two bodies is greater than that between two other bodies, that one body is hotter than another, by what are called qualitative tests. As we grow older and more mature, we find that it is necessary to know exactly how much heavier one body is than another, and exactly how far one body is from another. The parts of a steamship must be fitted together with the utmost nicety, otherwise the ocean never could be crossed. Whenever we make careful measurements of anything, our measurements cease to be qualitative and become quantitative. In qualitative tests of the nature of things we do not need accurate standards of length or weight. We can ascertain without a balance that lead is heavier than cork. We can pace off a certain dis-

tance and discover that it is greater than another distance without actually knowing how long one of our paces is. If we wish, however, to ascertain whether one coin is gold and another half gold and copper, we must make quantitative measurements, and need a standard of weight. If we wish to order a carpet for a room, we must do more than pace it. We must have a foot-rule or a metre-scale, and we must measure it exactly.

Our increase in knowledge can therefore be tested by our care in making measurements. The difference between the scientific man and the unscientific man is that one makes accurate measurement while the other is satisfied with general knowledge and does not ask the question, "How much?" In order to answer the question "How much?" in regard to distance, we must settle upon some unit which will not change with time, which will be the same a hundred years from now as it is to-day. The paces of a man alter from year to year, and different men have different paces. It is very important to the advance of science that the measure of length should be invariable in all countries. The French measure of length is called a metre, and is about three feet, or exactly 3.280 feet. The advantage of using the metre is that it is divided into decimal parts—a decimetre is a tenth of a metre—a centimetre, one-hundredth of a metre—a millimetre, one-thousandth of a metre—so that to reduce, for instance, 3,600 millimetres to metres you have merely to move the decimal-point three places to the left, or, in other words, divide by one thousand. To reduce inches to feet, in English measure, it is necessary to divide by twelve, and this takes

time and mental effort. There is, therefore, economy in the use of the French or decimal system of measure of length.

EXPERIMENT 1.—Ascertain the average length of one of your paces by pacing a certain distance and then measuring it and dividing the distance by the number of your paces. Then pace a room to ascertain how many yards of carpet one yard wide will be necessary to cover it. Suppose the carpet is three dollars a yard. Estimate its cost.

EXPERIMENT 2.—Measure the length and breadth of your room in both French and English measure.

Instructions.—Record each measure as follows, in fractions of a centimetre and also of an inch:

Measures of

Length. Cm.	Length. Inches.	Breadth. Cm.	Breadth. Inches.
....	....	....	...
....	....	....	....
....	....	....	....
....	....	....	....
Mean length = ...	Mean length = ...	Mean breadth = ...	Mean breadth = ...

The mean or average length is obtained by dividing the sum of the observations by the number.

Calculate the expense of carpeting, and compare the results of Experiment 1 and Experiment 2.

EXPERIMENT 3.—Measure the circumference of two circles, and find the number you must multiply the radius of the circles by in order to obtain the circumferences.

Instructions.—Draw two circles of different radii upon stiff paper. Cut these circles out carefully with scissors, and, having drawn a straight line upon the page of your note-book, cause the cir-

cles to roll along the straight line and measure the spaces passed over. These spaces are evidently the circumferences of the circle. Measure the radii of the circles and divide the length of the circumferences of the circles by the values of the radii. In this way you will obtain the number by which you must multiply the radius of any circle by to obtain the length of the circumference. This number is generally written 2π, in which $\pi = 3.1416$.

Hence, one half of the number you obtain experimentally should be equal to 3.1416.

Having settled upon our unit of length, we can determine how many cubic metres or feet could be placed in a given room. A cubic foot measures one foot in breadth, one foot in length, and one foot in height. A cubic metre measures one metre in breadth, one metre in length, and one metre in height. Moreover, one cubic foot contains 1,728 cubic inches, and one cubic metre, 1,000,000 cubic centimetres. Knowing the volume of water which certain standard vessels contain, we can ascertain the volume of any irregular body. It is found most convenient to adopt as unit of volume a cubic centimetre—that is, the volume occupied by a cube, each side of which is a centimetre, or one-hundredth of a metre.

EXPERIMENT 4.—Measure the volume of water which two hollow cylindrical vessels will contain in quarts and in litres, also in cubic inches and cubic centimetres.

Instructions.—One vessel graduated to cubic centimetres is necessary. From a table of French and English measures calculate the number of cm. in a litre and in a quart.

EXPERIMENT 5.—Compare the volume of two cylinders of the same height.

Instructions.—Proceed as in the previous experiment, and obtain the volumes in cubic cm. Since the cylinders have the same height, the variation in the volumes must be due to the radii of the bases of the cylinders. Measure the radii of the bases, and see whether the volumes of the cylinders vary as these radii, or as the square of these radii. Then find by trial the value of the number C by which you must multiply the radius; or the radius squared in order to obtain the area of the bases of the cylinders. The volume of a cylinder is equal to the area of its base multiplied by its altitude. Prove this by your measurement, and compare the value of the number C with the value of π obtained in Experiment 2.

EXPERIMENT 6.—Measure the volume of an irregular piece of stone.

Instructions.—Suspend the stone by a fine string or wire in a glass vessel filled with water to a certain height, so that the displacement of water by the stone will cause the water to flow out of the vessel through an orifice. Measure the amount of water that overflows, and this will give you the cubic contents of the stone.

CHAPTER II.

SPECIFIC GRAVITY.

OUR experiments thus far would enable us to measure distances, and to ascertain the volume or space occupied by bodies. Besides the ideas of space, which we obtain first by rough estimates and afterward by careful measurements with a standard of length, we obtain ideas of the relative weight of bodies. We perceive by our muscular sensations that it is harder to lift one body than another, and we could obtain equal volumes of different substances and compare the muscular sensations we should have in lifting these equal volumes in comparison with lifting an equal volume of water. To perceive how erroneous such a qualitative test would be, it would only be necessary to blindfold a person and give him the unit of volume made of different substances and ask him how much more or less they weighed than an equal volume of water. Long practice might give a certain skill, but no one would choose to have his gold or silver estimated in this way. We could, however, take the weight of a cubic centimetre of water at 0° C. or 32° Fahr., since this is a fixed temperature, and compare the weight of other bodies with this standard.

EXPERIMENT 7.—Coil a fine steel wire around

a smooth cane or other cylinder, so that, when the wire is slipped off the cylinder, it shall form a spiral. Suspend this spiral vertically in front of a long piece of looking-glass. Arrange a bit of wire upon the spiral in any suitable position, so that it may indicate by its movement up or down the change of weight of a body that is put in a little scale-pan, which is attached to the lower end of the spiral. The object of the mirror is to prevent the error which would arise from the different position of the observer's eyes. When the pointer covers its reflection in the mirror, the eyes, the pointer, and the reflection are all on the same line. With this instrument compare the weight of a cubic centimetre of water, which weighs one gramme, with that of a cubic centimetre of alcohol.

NOTE.—In the first place, test the accuracy of the spring with different weights, and be careful not to permanently stretch the spring.

EXPERIMENT 8.*—You are given six straight wires and one twisted one, all cut from the same piece, and are required to find, by measuring and weighing the straight pieces, the average weight of a centimetre of the wire, and thence to calculate the length of the twisted piece.

Instructions.—The observations are to be recorded in the following form.

Finish all the weighing and measuring before doing any calculation.

The calculated weight W′ of each wire is found by taking the mean value of the weight of one centimetre and multiplying this by the length.

*** From Professor Hinrich's "Elements of Physics."**

NO. OF WIRE.	L, length in cm.	W, weight in grammes.	$\frac{W}{L}$, weight of 1 cm.	W, calc. wt. of wire.	E, error of W–W.′
No. 1.					
No. 2.					
No. 3.					
No. 4.					
No. 5.					
No. 6,					

Mean, ...

Bent wire, ... Weight, ... Calc. length, ...

EXPERIMENT 9.—Obtain the specific gravity of the piece of stone of which you have obtained the volume in Experiment 6.

Instructions.—The specific gravity of a body, or its density, is its weight compared with the weight of an equal volume of water. Since, in French measure, a cubic cm. of water weighs 1 gramme at 3.9° C., if one should find that the stone weighs 100 grammes, and has a volume of 10 cubic cm., each cubic cm. will weigh 10 grammes, or 10 times as much as a cubic cm. of water, and hence the specific gravity of the stone will be 10. You must therefore obtain the weight of the stone. To do this, suspend it from an ordinary spring-balance.

EXPERIMENT 10.—The upward pressure on a body immersed in water is equal to the weight of the volume of water displaced.

Instructions.—Suspend the stone used in Experiment 6 by a fine string from the hook of a spring-

balance, and immerse the stone in water. Compare the weight indicated by the balance when the stone is weighed in air and afterward in water, and compare the loss of weight with the weight of the water which was displaced by the stone (Experiment 6).

EXPERIMENT 11.—Find the density of acohol by weighing in the liquid.

Instructions.—Suspend the stone used in Experiment 6 from a spring-balance and immerse it in a vessel containing alcohol. Observe the loss of weight, and compare it with the loss of weight of the stone when it is immersed in water. Since the loss of weight is the weight of an equal volume of liquid, we can compare the weights of the equal volumes of water and alcohol, and this comparison will give us the specific gravity of the alcohol.

The advantage of the term "specific gravity" can be perceived from the preceding experiments. It tells us what we must multiply a volume of water equal to that of the given substance by in order to obtain its weight. The adoption of water as a standard of volume has great convenience, especially in the French measure, for the weight of one cubic centimetre of water at 3.9° C. is one gramme, and, when we know the volume of any vessel, we also know what weight of water it contains, and if it contains any other liquid we can also obtain its weight by ascertaining its specific gravity. The specific gravity of a man is not far from that of water; therefore, a man should be able to float in water with very slight exertion.

There is a large class of instruments called hydrometers which determine the specific gravity of a liquid by the degree to which they sink in this liquid

compared with their depth in water. It is very easy, for instance, to attach a weight to a hollow tube closed at one end so that it will float vertically in water at a certain depth, and at another depth in pure alcohol, and to graduate the distance between the lines which mark these two depths, so that the graduation shall give the percentage of alcohol in a mixture of water and alcohol. These are relative instruments—that is, they give relative indications.

EXPERIMENT 12.—Subdivide the tube of a hydrometer so that the scale will indicate percentages of alcohol in a mixture of alcohol and water. The word *subdivision* is used to denote the operation of dividing a scale into portions which shall represent —for instance, in this case—the immersion of the hydrometer-tube with different percentages of alcohol. Thin glass tubes suitable for hydrometers can be readily obtained. An ordinary piece of glass tubing can be drawn out at one end into a bulb; in this bulb can be placed a suitable number of fine shot, in order to make the tube float upright. A paper scale can be pasted inside the tube, and the value of its scale divisions, in percentages of alcohol, can be entered in a note-book. These percentages can be made by mixing known volumes of alcohol with known volumes of water.

We can employ water in general as a medium by which the weights of different bodies can be compared. It is only necessary to place the body to be weighed upon a float and see how far it depresses the float, and then replacing the weight by known weights until the same point of immersion is reached. The weight of the body will then be

equal to the weights which produced the same immersion. Attaching the body afterward beneath the float in the water and placing weights upon the float until the same fixed point of immersion is reached, we know that these weights balance the upward pressure of the water which the body displaces, and are equal to the number of cubic centimetres in the volume of the body.

EXPERIMENT 13.—A hydrometer, called Nicholson's, can be readily made by any tinman. It consists of a cylindrical float, to the top of this is attached a stout wire that supports a plate upon which the weights and the body to be weighed can be placed. At the other extremity of the float is attached a little scale-pan in which the body can be immersed in the liquid. The float is weighted so that it remains upright, and sinks to a certain mark *c* upon the wire.

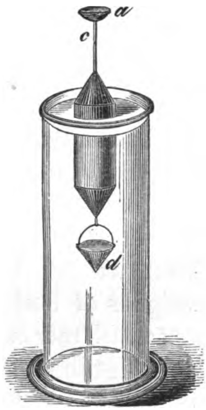

FIG. 1.

The body is first placed upon the plate with additional weights in order to sink the hydrometer to a fixed mark upon the stem. The body is then removed, and the increase of weight necessary to again sink the hydrometer to the same mark will give the weight of the body in air. The latter is then placed in the lower scale-pan. The weights in the upper plate, or pan, necessary to sink the hydrometer to the same level, will be greater than when the body was placed in the upper pan. The difference of the weighing of the body in the two

pans gives the loss of weight of the body in water, or, in other words, the upper pressure of the water. This upward pressure is equal to the weight of the water displaced by the body. Knowing this weight in grammes, we know the volume of the body in cubic centimetres, and dividing the weight in air by the loss of weight in water will, therefore, give us its specific gravity. If we call W the weight of the body in air, and W_1 its loss of weight in water, we have sp. gr. $= \frac{W}{W^1}$.

Our measurements of weight and volume thus far have been conducted without reference to the air about us. This air, however, has weight. We are immersed in it, together with all our apparatus. It is as if we were at the bottom of a sea, and were conducting our measures of weight with reference to a lake of mercury, which answers to the water when we are in the air. We might compare the weights of bodies with the weight of a cubic centimetre of mercury, and obtain our specific gravities by weighing the body in the water which surrounds us instead of the air, and afterward observing the loss of weight in the mercury. Suppose, however, that the glass vessel with which we make our measurements of volume should contain air instead of water, it is evident that it would not weigh as much as an equal vessel filled with water. In making our calculations we should have to make allowances for the weight of water which fills the vessel, for we are balancing this vessel, with its contained water, against certain weights which do not displace the same volume of water as the vessel.

Above the water and in our atmosphere we are

also surrounded by a sea of air which exerts pressure just as the water does when we are beneath the ocean. The common experiment of pressing a disk of wet leather against the floor, and endeavoring to pull it away by a string which is attached to its middle, illustrates the weight of the atmosphere. In endeavoring to pull up the disk, we are endeavoring to lift a column of air which extends from the floor to the extreme limits of our atmosphere. This column has a weight equivalent to fifteen pounds on every square inch. If our disk were just one square inch, and, if we should attach a spring-balance to the string which is connected with the disk, the balance would register a pull of fifteen pounds just as the disk is pulled from the floor. The higher we go up in our atmosphere, the less pull we should have to exercise. It is evident, therefore, that if we make delicate weighings we must take into account the weight of the air in the vessel whose volume we wish to measure.

Our experiments hitherto upon the relative masses of two bodies have been performed by the aid of given weights. Now, the weight of a body depends upon the pull of gravitation upon every particle in the body. Bodies differ from each other in weight because there is more or less matter to be attracted to the earth, and to also exercise attraction upon the earth, in the one than in the other. The force of attraction, or the pull of gravitation, varies at different places on the globe. A pound of tea at Canton is attracted with a certain force. When we carry this pound of tea and measure the pull of gravitation upon it in Newfoundland we find that it weighs more than a pound. If, however,

in Newfoundland we balance it against the same weights that we used at Canton, it will still balance with a pound-weight, for the pull on the weight is the same as the pull on the pound of tea. The force of attraction between the particles of the tea and the earth and the particles of the standard weights and the earth, is less at Canton than in Newfoundland, yet the amount of matter in the volumes of the two substances remains the same. In our operation of weighing we really compare, therefore, the amount of matter in two volumes. This is what we call mass of the bodies.* We determine by weighing the masses of two bodies and not their absolute weight, or the force by which they are attracted to the center of the earth. In order to determine the weight, we must know the exact value of the force of attraction of the earth at the place where we make our experiment. This force has been carefully determined at the latitude of Paris, and our gramme-weights will represent at Paris the absolute weight of one gramme; in any other latitude one-gramme weight has not an absolute value, and we must correct it by ascertaining the ratio between the force of gravitation at that latitude and at the latitude of Paris. In practice, however, it is not necessary to know the absolute weight, and in weighing we determine the masses of bodies. The methods that have been given in this treatise for determining the masses of bodies are sufficiently accurate for many purposes; but when it is necessary to determine the mass of a body (what is commonly termed weight) to the hundredth and thousandth place of decimals, or, in other words, to centigrammes and to milli-

* Mass will be defined by reference to inertia later.

grammes, we use the chemical balance. This is explained in all text-books of physics, and it is not necessary to enter into a full description here. In its elements it is a horizontal bar balanced upon a knife-edge. It has also a knife-edge at each extremity, upon which are hung the scale-pans. A pointer shows when the balance-arm is horizontal, or, in other words, when the masses in the two scale-pans balance each other. We test the equality of the length of the arms of the balance by placing a body first in one pan and then in the other, and see if the same weights are necessary to bring the balance into a horizontal position in the two cases. The center of gravity of the balance-arm should be slightly below the knife-edges. If it is too low, the balance loses in sensitiveness; if too high, the balance becomes unstable; and if it is on the line with the knife-edges, the balance-arm will not return to its horizontal position after it has been disturbed. The three knife-edges must be in line. These precautions, it will be observed, arise from the action of gravitation upon the particles in the balance-arm. When the particles are equally distributed along the balance-arm, the pull of gravitation on one side of the middle of the balance-arm is equal to the pull on the other side. A pointer is attached to the balance-arm, and this should point to zero on the little scale attached to the balance. When only moderate accuracy is needed, we can take the mean of two successive points at which the pointer or index-hand turns when the balance-arm is swinging, and take the mean of these for the zero-point. If we can not make the pointer return to a zero-point when we are weighing, on account of want of small

weights, we can observe how much the pointer moves for a centigramme or a milligramme, and determine how much of this space the pointer lacks from the zero when the balance is unloaded. One should never weigh while standing, and the directions in the following experiment should be carefully noted.

EXPERIMENT 14.—The general precautions are as follow: The object, if perfectly dry and non-corrosive, can be placed directly in the scale-pans. If it will attack the scale-pan, it is usual to place it in a watch-glass or other protecting vessel, which is then rested upon the scale-pan. The balance-arm being off its knife-edges, place one of the weights in the other scale-pan, and carefully turn the milled head so as to lower the balance-arm upon its knife-edge. This must be done very carefully and slowly. If the beam falls suddenly upon the knife-edges they are apt to be ruined. When large weights are used, a very small amount of lowering will show which pan of the balance contains the larger weights. The smallest weights must not be touched with the fingers; a pair of forceps is always provided for them. Weights should be deposited either in the scale-pan or in their proper places in the box. If they are placed on the table, they become dirty, and are generally swept off by the arm of the weigher and lost. To save time in weighing, proceed as follows: Place the body in one scale-pan, and, for instance, a 6-gramme weight in the other. The pointer moves toward the 6 grammes. This shows that 6 grammes is too little. Replace by twice 6 grammes or 12 grames. This is too much, and the pointer moves in the opposite direction, halve the additional 6 grammes—that is, add 3 grammes, making in all

9 grammes. This is too light. Add one half of 3 grammes, or one gramme and five decigrammes, making in all 10.5. This is too heavy. Subtract seven decigrammes; this is still too light. The weight is between 9.8 and 10.5. Proceed in the same way, and do not add small weights by guess, for much time will be lost. Make a set of small weights out of tin-foil by weighing a rectangle of the foil of a suitable size, and, having computed its area by multiplying its length by its breadth, cut it up into parts which will represent proportional parts of the ascertained weight of the rectangle. The weights of these parts thus found by calculation can be compared with their actual weight in the balance-pan. It is well to turn up one corner of these weights in order to take them up readily with the pincers. The amount of weight they represent can be punched upon them.

By means of a delicate balance we can repeat many of the preceding experiments and obtain closer results. By attaching the body, whose specific gravity we wish to obtain, by means of a fine string to a hook beneath one scale-pan of a balance, and weighing the body in water, and afterward in air, we can obtain its specific gravity by dividing its weight in air by its loss of weight in water. A balance adapted for this is called a hydrostatic balance.

EXPERIMENT 15.—Determine the specific gravity of lead by the hydrostatic balance. Allowance should be made for the fine wire by which the body is attached to the bottom of one of the scale-pans. If it is moistened with a little spirits of wine, the pull of the water upon it can be very much lessened.

EXPERIMENT 16.—Determine the specific gravity of lead-shot by a specific-gravity bottle. Obtain of the druggist a glass vial with a ground-glass stopper. The capacity of the vial should be about 30 cubic centimetres. Determine the capacity by weighing the bottle first empty, and afterward full of water. The difference between these two weighings is the weight of the volume of water which the vessel contains; it will be expressed in grammes and decimal parts of a gramme; and the number of grammes will also give the number of cubic centimetres, since a cubic centimetre of water at 3.9° C. weighs one gramme. Then, having weighed a certain amount of the lead-shot, pour out some of the water of the flask and pour in the shot. Shake the bottle gently to drive out the air-bubbles, fill up with water, and weigh again.

Let W = weight of shot in air.
" w = " " bottle and water.
" W_1 = " " bottle, water, and shot.
" d = density of water at the temperature of the experiment.

The weight of the water displaced is $W + w - W_1$; hence the specific gravity $= \delta = \dfrac{dW}{W + w - W_1}$.

EXPERIMENT 17.—Determine the specific gravity of alcohol, and its percentage strength. Having ascertained the capacity of the specific-gravity bottle, pour out the water and carefully rinse the vessel with small amounts of alcohol several times. Then fill with alcohol and weigh again, taking care that the alcohol is at the same temperature as the water which we used to obtain the capacity of the bottle, to obviate corrections for variations in the capacity

of the flask, since a change of temperature would cause the bottle to expand or contract. The weight of the alcohol divided by that of the water will give the specific gravity of the alcohol.

In our determination of the specific gravity of bodies we speedily discover that the temperature of a body greatly influences our results. All bodies, with few exceptions, expand under the influence of heat. The point of rest of our spring-balances which were used in Experiment 7 will change with the temperature of the room. Air heated in a tightly-corked flask will burst the flask. A tightly-fitting glass stopper can often be removed by placing a hot wet cloth around the neck of the bottle. When a body expands, its volume is larger, although its mass is the same; it will, therefore, displace a larger amount of water. If, then, we divide the weight in air by the loss of weight or the volume of displaced water, at a temperature above zero, our specific gravity would be too small for the specific gravity at 0°. The greater the expansion of our substance, the less would be its specific gravity. Hence, to refer the specific gravity to 0° C., we must correct for the cubical expansion for each degree of temperature.

Let us see what the cubical expansion is. The coefficient of expansion of a long rod, for instance, under the effect of t would be found by (knowing the length of the rod at 0°) dividing the elongation expressed in fractions of a metre by the length of the rod in metres, and by the number of degrees of temperature above 0°. Or the volume at the temperature t would be l, the length at 0° plus the increase in length at $t°$.

We can express this increase as a certain part of l multiplied by the rise in temperature t or klt in which k is called the coefficient of expansion. Hence the volume at $t = l + klt = l\,(1 + kt)$. This is the formula for linear expansion. Let us take now the expansion of a cube. Each side of the cube expands to $l\,(1 + kt)$.

The volume of the cube is the product of its base by its altitude or $l^3\,(1 + kt)^3$. If we expand $(1 + kt)^3$, knowing that k is a very small quantity, we can neglect the squares and cubes of k, and can take $1 + 3kt$ as the value of $1 + 3kt + 3k^2t^2 + k^3t^3$. Hence $3k$, the coefficient of cubical expansion, is three times the coefficient of linear expansion. Let us place $3k = \beta$. Now we can reduce our values of specific gravity obtained at t to $0°$.

EXPERIMENT 18.—If sp. gr. and (sp. gr.)′ are the specific gravities of lead at t and t^1 degrees, and W the weight of the lead, V the volume at 0°, and β the coefficient of cubical expansion, we have the volumes at t and t'.

$$V(1 + \beta t) \text{ and } V(1 + \beta t^1) \text{ and}$$

$$\text{sp. gr.} = \frac{W}{V(1 + \beta t)} \quad (\text{sp. gr.})' = \frac{W}{V(1 + t^1\beta)}$$

$$\text{also } \beta = \frac{(\text{sp. gr.}) - (\text{sp. gr.})'}{(\text{sp. gr.})'t^1 - (\text{sp. gr.})t}.$$

This method, therefore, enables us to obtain the linear expansion, for this is one third of β.

CHAPTER III.

PRESSURE OF THE AIR.

WHAT is called the force of buoyancy arises from a difference of pressure. The upward pressure, for instance, on a surface at $c\,c'$ (Fig. 2) exceeds the downward pressure on $b\,b'$ by the weight of the column of liquid $b\,c\,c'\,b'$. What is often called the amount of buoyancy is the weight of the volume of fluid which is displaced by the body. What is erroneously called the force of suction is also a manifestation of the pressure of the atmosphere. We remove the pressure of the atmosphere from one end of a tube by drawing out the inclosed air, and, the pressure being thus removed at this end, the atmosphere forces itself, or any fluid, into the other end.

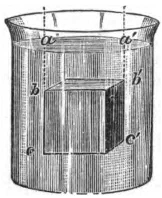

FIG. 2.

EXPERIMENT 19.—Bend a glass tube which is open at both ends into the form of a U, one arm being somewhat longer than the other; fill the U-tube with water or any liquid up to the level of the shorter arm. Close the end of the shorter arm tightly with the finger, and endeavor to suck out the liquid. Why do you not succeed?

EXPERIMENT 20.—In order to obtain the weight

of air, we can proceed as follows: Take a Florence flask of about 300 cubic centimetres, fill it one quarter full of water, place it over a sand-bath, and boil the water. The steam will expel the air in the flask. After the boiling has been conducted for some time, remove the flask from the sand-bath, and, when the boiling has ceased, loosely cork the flask and cool it by putting a wet cloth around the upper portion. This will cause the water to boil again. Then tightly cork the flask with a rubber cork, and, the temperature having become that of the room, weigh the flask and the water that remains in it. The peculiar water-hammer sound that the flask gives when shaken will show that the air which the flask held has been expelled. Then take out the cork and weigh the flask again. It is now filled with air and water, the amount of water being the same in both weighings. The difference in the weighings will give the weight of the number of cubic centimetres of air above the water in the flask. Knowing the capacity of the flask, we have the weight of one cubic centimetre of air (in contact with water). The weight of one cubic centimetre of dry air at 0° and at 760 millimetres pressure is .001293 of a gramme. A very good approximation to the true value can be obtained with ease in this experiment. In tightly corking the flask, it is well to protect the hands by means of a thick towel from the possible breakage of the flask.

EXPERIMENT 21.—Close a glass tube at one end, grind the edges of the other end upon emery-paper or upon a grindstone, so that the tube may be closed tightly by means of a piece of glass or a sheet of cardboard; fill the tube with water and

place the glass or cardboard over the open end, and invert the tube. The water will not run out. The pressure of the atmosphere, or, in other words, its weight upon the end of the tube, holds the weight of water in the tube. In this way, a tube thirty feet long, filled with water, could be supported with merely a thin plate upon the open end to distribute the pressure equally.

EXPERIMENT 22.—Close the end of a stout glass tube (see Appendix), which is about one centimetre in internal diameter and 80 centimetres long, and fill it with mercury. Place the thumb closely over the open end, and invert it in a vessel filled with mercury. The mercury-column will stand at a height of about 760 millimetres. The weight to the square inch of this height of mercury will be found to be fifteen pounds, or about seven kilogrammes. This inverted tube filled with mercury constitutes a barometer.

Let us not forget that we are at the bottom of an ocean of air, and that all the experiments on weight and pressure of water are equally applicable in the ocean of air.

It has been shown by Andrews and others that by sufficiently great pressure and at sufficiently low temperature we can compress all gases into the liquid form, and that there is a continuous gradation from the state of a liquid to the state of a gas. A beautiful experiment consists in inclosing a certain amount of carbonic-acid gas in a little glass cylinder, and then, by great pressure, showing that the line of separation between the liquid produced and the carbonic acid left above the liquid is a shadowy one, and that the liquid fades insensibly into the gas.

The simple experiment of blowing soap-bubbles shows that the pressure of a fluid is exerted equally in all directions. If this were not so, the bubble would not be spherical.

EXPERIMENT 23.—Pour a small amount of liquid glycerine soap (see Appendix) into a shallow vessel. Connect a tobacco-pipe, by means of a piece of rubber tubing, to a gas-jet, and allow the gas (street or hydrogen gas) to flow into the pipe. In this way bubbles filled with hydrogen gas can be formed. The weight of the hydrogen gas is less than that of the atmosphere, and these ascend, just as a thin bulb of glass filled with air would ascend from the bottom of the sea.

The disturbances in the atmosphere are shown by the variations in the height of a barometer. When the crest of a wave is over a barometer, there is greater weight of air, and consequently it takes a greater height or weight of mercury to balance it; the depression of the wave is followed by a low barometer.* The weight of the air depends upon the amount of moisture it holds. The more moisture it holds the heavier it is. Dry, warm air can take up more moisture than cold. Besides the actual weight of any fluid, we find that the pressure due to this weight upon a body immersed in a fluid is equally exerted in all directions.

A man does not feel the weight of the atmosphere, for the pressure within him at all points is equal to that on the outside of his body.

EXPERIMENT 24.—Bend two glass tubes as in Fig. 3; pour a certain amount of mercury into

* The upward and downward movements of air also affect the barometer.

them and immerse them in water. The difference in height between the surfaces of the mercury-columns gives the pressure of the water upon the ends of the tube. It will be noticed that the vertical pressure of the water in one tube is equal to its lateral pressure in the other, and this will be true as we turn the tubes about a vertical axis, keeping their orifices always at the same level. By immersing either tube farther, we find that the column of mercury supported is greater, and therefore that the pressure increases with the depth. This is necessarily so, since the pressure is simply the weight of a column of water resting upon the lower surface of the mercury.

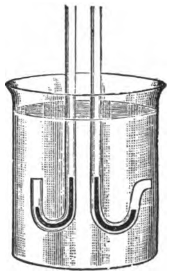

FIG. 3.

This fact, when we reason upon it, shows that the pressure on the bottom of any vessel, which is immersed in a fluid, depends merely upon the area of the base of the vessel and the depth of each element of the base below the surface of the fluid.

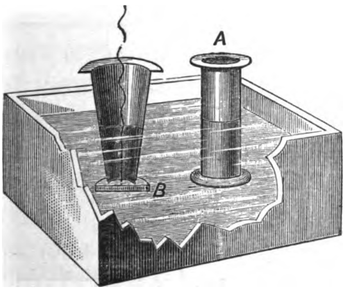

FIG. 4.

EXPERIMENT 25.—Obtain from a tinman a cylindrical tube, *A*, and a tin vessel made in the form of the frustum of a cone, both vessels being open at each end (Fig. 4), the end *A* being equal to the end *B*, with

a small flange of tin at one end. Cover this end with a well-fitting piece of leather, to the middle of which is attached a string. Immerse the vessels in succession to the same depth, and pour in water until the leather is released. This will happen when the level of the liquid in the vessel is equal to the level of the water outside. But the pressure on the end *A* of the cylinder is equal to the weight of the column of water resting upon it. *A* and *B* being at the same depth, and having the same area, the pressure is the same upon them, and is independent of the form of the vessels. By inverting the vessel *B* we can prove that the pressure on the base can be greater than the weight of water which it can hold.

This behavior of fluids shows that we can lift very heavy bodies by the exertion of a very small pressure.

For, suppose we exert a pressure of a kilogramme upon a piston through one centimetre, and communicate by means of a fluid this pressure to every unit of area of the larger piston. The pressure multiplied by the distance passed through is the work we exert. The piston *P* (Fig. 5) moves up a certain height under the influence of the pressure we exert on the smaller piston. We shall therefore have the pressure p on the small piston multiplied by the distance through which it is exerted, or s equal to

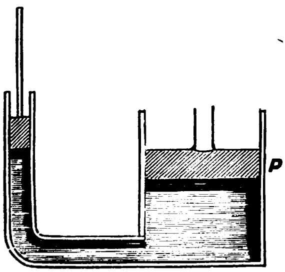

FIG. 5.

the same pressure p on each unit of area A of the base of the larger piston P multiplied by the distance through which the larger piston moves; or:—

$$ps = pAs'$$

in order to make these two values of work equal s' must be very small compared with s.

In order to keep the level of water constant, the following simple contrivance is used, which also illustrates the pressure of the atmosphere:

EXPERIMENT 26.—Fill a small bottle (Fig. 6) and invert its nozzle under the water whose level we wish to keep constant. As the water in the lower vessel evaporates, the nozzle of the suspended bottle is uncovered, and a bubble of air enters and drives out water enough to cover the nozzle again to the same amount.

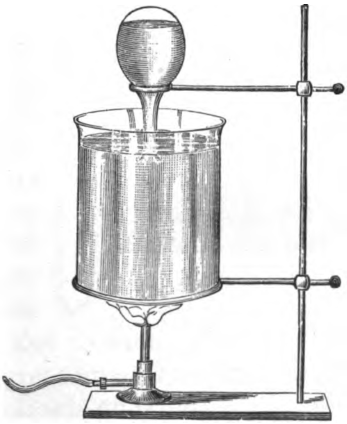

FIG. 6.

In order to drive out the air from a vessel, we can exhaust it by means of an air-pump. Instead of using pistons, the modern laboratory-pump employs the mechanical action of falling water or mercury.

EXPERIMENT 27.—Cut a thick bottle in two (see Appendix). Fit it tightly with a cork in which a short tube of glass is inserted. To this tube join hermetically tight a short piece of rubber tubing,

upon which is slipped a pinch-cock, *P* (Fig. 7). Connect this rubber tubing with a three-way joint, *J*, of glass or of iron gas-tubing. Insert in one of the ways of this joint a piece of glass tubing about a metre long; one end of this dips in a mercury-vessel, *M*. Connect the vessel to be exhausted by a short piece of glass tubing with the other way of the joint. Fill *A* with mercury and allow the mercury to fall slowly through the long tube, the internal diameter of which should not be more than three millimetres. The mercury will fall in detached portions, separated by air-columns, which come from the vessel that we are exhausting. This air rushes out to fill the more or less rarefied portions of the vertical tube, the mercury in falling having displaced the air. The amount of exhaustion is shown by the height of the mercury-column above the level of mercury in the lower vessel *M*. The Florence flask *F* can be removed by applying a jet of gas at *a*.

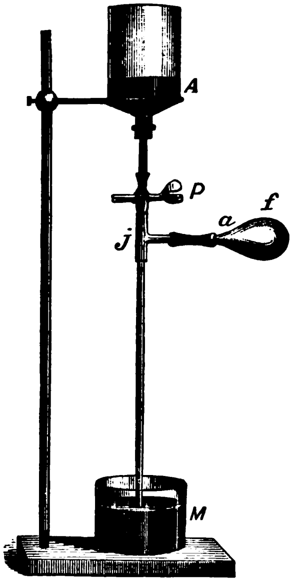

FIG. 7.

Where a head of water can be obtained, this can be substituted for a head of mercury. An apparatus called Richards's aspirator can be screwed upon a water-faucet, and is very useful as a substi-

tute for an air-pump. In this instrument, the velocity of water flowing through a narrow orifice serves to drive down small columns of water separated by columns of air; the weight of mercury accomplishes the same object in Experiment 27. (See Appendix.)

No matter what medium is used in the pump, it is usual to employ the apparatus shown in Fig. 8. The fluid enters by the tube *a*, and falls through the narrowed portion of the tube *b*. The tube *T*, coming from the vessel which we desire to exhaust, is provided with a tube, *U*, which contains mercury, and serves to show how far the exhaustion has proceeded. Instead of allowing the water to fall through *a*, it can be blown through, or caused to issue with considerable velocity, and thus inclose bubbles of air and draw them down with it. The form of the tube at *b* is found to be the best for entangling the air-bubbles. The pinch-cock *S* regulates the supply of mercury or water.

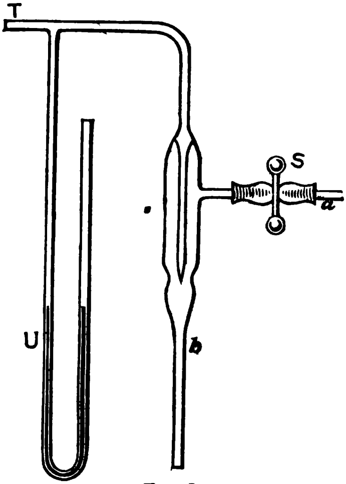

FIG. 8.

EXPERIMENT 28.—Measure the relative density of mercury and water by means of the atmospheric pressure.

Bore two holes in a cork. Bend two glass tubes

twice at right angles, and fit them into these holes in the cork, arranging these glass tubes so that when the cork is fitted to a bottle or flask the portions of the glass tubes that are outside the bottle can dip into vessels containing the mercury and the water. Heat the air contained in the bottle A portion of the air will be expelled in bubbles through the liquid. Then allow the bottle or flask to cool, and measure the relative heights of the two liquid columns. The column of water will be found to be 13.59 times that of the mercury. This number will be the specific gravity of the mercury. Repeat the experiment with alcohol instead of mercury. The cork must fit the flask tightly.

EXPERIMENT 29.—Ascertain the pressure of air in a flask from which the air has been partly expelled.

Allow both of the tubes inserted in the apparatus described in Experiment 28 to dip into vessels containing mercury. Heat the air in the flask until a certain portion of it is expelled in bubbles through the mercury. Then allow the flask to cool. Measure the heights of the mercury-columns, and compare with the height of the mercury-column in Experiment 22, which gives the atmospheric pressure.

EXPERIMENT 30.—Verify Boyle's or Mariotte's law.

This law states that the volume of air contained in a closed tube decreases in proportion as the pressure upon the air is increased. Calling the pressure P and the volume V, we have $P \times V =$ a constant quantity. Close a glass tube, of about one metre in length, at one end. Paste a thin strip of paper along the tube, beginning at the closed end.

Graduate the tube in cubic centimetres by pouring in mercury from a vessel graduated to cubic centimetres. This operation is called calibration. Having finished the calibration, pour in mercury until the tube is about half full. Putting your thumb over the open end of the tube, invert it in the receptacle (see description of receptacle below). Then lift the tube through varying heights and measure the column of mercury in the tube, and also the volume of air above the mercury. The column of mercury and the pressure of the inclosed air evidently balance the pressure of the atmosphere. Hence, subtracting the heights of the mercury-columns from the height of the mercury-column in Experiment 22, which measures the pressure of the atmosphere, we obtain the pressure of the inclosed air. Then multiply these values by the corresponding volumes. Take four different observations, and arrange the results thus:

$$
\begin{aligned}
P_1V_1 &= \ldots. \\
P_2V_2 &= \ldots. \\
P_3V_3 &= \ldots. \\
P_4V_4 &= \underline{\ldots.} \\
\text{Mean} &\ \ \ldots.
\end{aligned}
$$

(*Receptacle.*—This can be made out of a piece of iron gas-pipe about 1 metre long and 2 centimetres internal diameter. This pipe should be closed by a nut at one end. Then cut a bottle in two (see Appendix), and fit the mouth of the bottle on the open end of the tube by means of a cement (see Appendix). This receptacle should then be mounted upright at the side of your working table.)

A practical application of Boyle's law is called a volumometer.

EXPERIMENT 31.—Instead of the barometer-tube closed at one end, used in Experiment 22, terminate an open tube by means of half a bottle, the edges of which have been carefully ground (see Appendix), so that it can be closed hermetically by a ground-glass plate, P (Fig. 9). First immerse the glass tube, when the plate P is removed, until the level of mercury inside at d is the same as that outside. Afterward put on the plate P, so that the apparatus is hermetically closed, and lift the apparatus. The mercury will then rise to some point, d^1. Call the height $d^1 = h$. The original volume V of the air, which was under the pressure of the atmosphere, is now increased to $V + v$. Calling the pressure of the atmosphere at the time of our experiment P, we shall have $\frac{V}{V+v} = \frac{P-h}{P}$. Let us now place a body, B, whose volume we wish to find, in the apparatus. It is evident that it will displace its own volume of air. We repeat our observations, calling the volume of the body $B = x$.

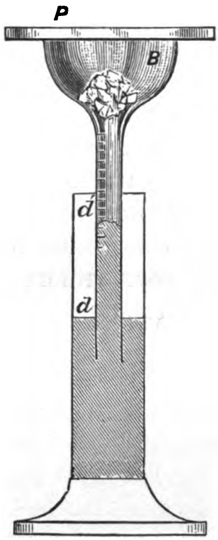

FIG. 9.

We shall then have $\frac{V-x}{(V-x)+v} = \frac{P-h_1}{P}$. From this and the preceding equation, we get $x = (V + v)$

$\left(1 - \frac{h}{h_1}\right)$. We can determine the volume $V + v$ by making the experiment with a cylinder of known volume. It is, of course, necessary to close the nozzle of the bottle, where the glass tube enters, perfectly air-tight; and to be sure that the plate P also closes the apparatus hermetically.

The law of Boyle is found to be only approximately true for very high pressures; for ordinary experiments in laboratories, however, it can be taken to be sufficiently exact. It is evident that all observations on the volumes of gases must be made at the same pressure of the atmosphere, or, in other words, at the same height of the barometer, in order to be compared. We can reduce our observations made at different barometric heights to the same standard by Boyle's law, for, since the volumes are inversely as the pressures, we have $\frac{V}{V_1} = \frac{P_1}{P}$, and, if we know the volume V at the pressure P, we can obtain V_1 at the pressure P_1.

If we wish to obtain also the density of a gas in terms of its density at a standard pressure, we proceed as follows:

EXPERIMENT 32.—Let M be the mass of the gas; V is volume, and D its density at one pressure, and V_1 and D_1 the corresponding values at another pressure; then $VD = M = V_1D_1$; or $\frac{V}{V_1} = \frac{D_1}{D} = \frac{P_1}{P} = \frac{H_1}{H}$; and, if we take $H^1 = 760$ millimetres as the standard pressure, we have $D = \frac{D_1H}{760}$ where D_1 is density at standard pressure.

EXPERIMENT 33.—Boyle's and Mariotte's law for pressures greater than one atmosphere.

Close a long glass tube at one end. Then bend this end into a U-shape, making one branch of the U much longer than the other. Graduate the shorter branch in cubic centimetres, by pouring in mercury and moving the tube so as to allow the imprisoned air to escape. Having graduated the tube, allow the mercury to run out, and then refill until the mercury stands at the same level in both branches of the U-tube. The pressure of the inclosed air is then the same as that of the atmosphere. Now pour in mercury into the longer branch and measure the height of the mercury in the longer tube, above the level of the mercury in the shorter tube. This mercury-height measures the pressure of the imprisoned air. Knowing the volumes of the imprisoned air and the corresponding mercury-heights, apply the same calculation as in Experiment 30.

The law of Boyle, which our experiments illustrate, is extremely important, for by means of it we can calculate the volume of a gas when it is subjected to different pressures, if we know its volume at some standard pressure. For instance, if we call the volume of a gas at 0° C. and 760 mm. of mercury pressure V, and we wish to know its volume V_1 at a pressure of $P_1 = 765$ mm., we have $\frac{V}{V_1} = \frac{765}{760}$ $V_1 = \frac{V\,760}{765}$; that is, we multiply V by the ratio of the barometric heights $\frac{760}{765}$. We perceive that, however we may change the pressure P^1, the product

of $V_1P_1 = V\,760$; that is, V_1P_1 is always equal to a constant quantity $V\,760$.

Now, there is another law, called the law of Charles, which states that "the volume of a gas under constant pressure expands when raised from the freezing to the boiling temperature by the same fraction of itself, whatever be the nature of the gas."

EXPERIMENT 34.—Take a glass tube about 16 cc. long, open at both ends, and draw into it, by properly inclining it, a little index of mercury about 10 mm. long. Move the index until it occupies a position near the middle of the tube. Then close one end of the tube by holding the end horizontally in a Bunsen burner and turning it quickly between the fingers. Mount this tube vertically upon a piece of graduated wood or ivory, such as is often provided with thermometer-scales, place the closed end in melting ice, and note the position of the index. Then suspend the apparatus in a Florence flask, or other receptacle, which is filled with steam, arising from water boiling in the lower portion of the flask. Read again the new position of this index. We can find in this way the ratio of the volume of air at 0° to that at 100°. Afterward remove the tube from the steam and observe the position of the index after the inclosed air has taken the temperature of the room. Find also the ratio between the volume of air at 0° and that at the temperature of the room. Find the amount of dilatation of the air for each degree between 0° and 100°, and each degree between 0° and the temperature of the room. The amount of dilatation should be $\frac{1}{273}$ for each degree centigrade, or the air in expanding from 0° to 100° increases from 1 to 1.36

In this experiment we have supposed that the bore of the tube is uniform. It is necessary to ascertain the actual volumes of air in the tube for different positions of the index by means of calibration (see Experiment 30).

The accurate value of this amount of expansion is 1.3665. Now, if we combine this law of dilatation of gases with Boyle's law, we see that the increased volume at 100° C. must be equal to the volume at the standard temperature 0° C. multiplied by 1.3665. In order to understand clearly the use of this property of gases, let us examine the phenomenon more closely.

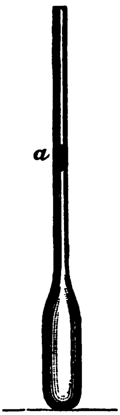

FIG. 10.

EXPERIMENT 35.—Blow a bulb at the end of a thermometer-tube of comparatively large bore (Fig. 10). Inclose a certain amount of air in the bulb and tube by means of a little index of mercury, *a*, as in Experiment 34. Place the bulb in melting ice, and notice the position of one end of the mercury-index. Afterward place the bulb in steam, and read the same end of the index, the pressure acting on the confined air remaining constant. It will be quickly seen that we can use this instrument to observe temperatures, instead of a mercury-thermometer. It constitutes what is called an air-thermometer. Supposing that the bore of the tube is uniform, we can graduate the space between the freezing and the boiling points into one hundred parts, and carry this graduation below the freezing, or 0° (Fig. 10) point, and above the 100°, or boiling-point. How far below the 0° point shall we go?

We know from our preceding experiment that the distance of the freezing-point from the bottom of the tube will be to the distance of the boiling-point from the bottom as 1 is to 1.3665. Or, if we call the volume of air in the tube when the index is at 0° V, and the volume when it is at 100° V_1, we shall have $\frac{V}{V_1} = \frac{1}{1.3665}$. If we call h (Fig. 11) the length of the tube of uniform bore from the bottom to 0°, the volume of air in this tube up to this point would be the section of the tube s multiplied by h or hs. The volume contained in the tube when the index is at 100° will in the same way be equal to $(h + 100)s_1$, since the distance from 0° to 100° is 100 scale divisions more than h scale divisions; we shall therefore have—

Fig. 11.

$$\frac{V}{V_1} = \frac{h\,s}{(h + 100)\,s} = \frac{h}{h + 100} = \frac{1}{1.3665},$$

we shall find that the value of h, which makes this equality true, is 272.85°, or, as it is generally taken, 273°. This point is called the absolute zero on the air-thermometer scale. The point 273° is merely a useful limit, and it is found convenient to reckon temperatures from this point, for at this point the air is supposed to be devoid of heat.

Experiment 36.—Take the apparatus of Experiment 33, and, having measured the volume of the gas in the U-tube at 0°, and the pressure upon it, which is the height of the barometer *plus* the difference in level of the two surfaces of mercury in the tube, submit it to the temperature of steam, and also change the pressure by a known amount by

pouring mercury into the longer arm of the tube; estimate this new pressure as before. We shall then find that, calling V the volume at 0°, and P the pressure at 0°, and V^1 the volume at 100° and P^1 the pressure at 100°, $\frac{VP}{V_1P_1} = \frac{273°}{273° + 100°}$; or, if we call the distance of the freezing-point and the thermometer reading from the absolute zero T and T_1, we have $\frac{VP}{V_1P_1} = \frac{T}{T_1}$, or $V = \frac{V_1P_1}{P}\frac{T}{T_1}$, which will enable us to reduce a gas to a standard pressure and a standard temperature, if we know its volume at any other temperature and pressure. A bent tube very much smaller than that used in Experiment 33 can be employed, so that the tube can be easily inserted in or placed over a vessel in which steam is generated.

CHAPTER IV.

MEASUREMENT OF DIMENSIONS.

IN our experiments thus far we have used principally a unit of mass, the gramme and its subdivisions. It is now necessary to investigate more closely the dimensions of bodies. It is not convenient to divide a measure of length much finer than into millimetres. There are about twenty-five millimetres to an inch, and it will be readily perceived that one twenty-fifth of an inch is a small quantity. Still, it is necessary to measure spaces much smaller than a millimetre. At first sight fine measurements seem a useless refinement, but as we study nature we find that we have to deal with extremely minute distances, which, although small in themselves, nevertheless influence our final results to a great extent. Take, for instance, the measurements conducted by the Coast Survey from which the area of a State is computed, and the positions of its great features are fixed by one line, which is called the base-line. This line is sometimes only a few hundred feet long. It must be measured with the utmost accuracy, for an error of a millimetre would make a large error in the line $E F$ of a triangle (Fig. 12), which is derived from the triangle $B C B_1$, of which $B B_1$ is the base. $E F$ may be several miles

long. An error of a millimetre in such a base-line might cause the center lines of two borings for a tunnel to diverge many feet.

In order to subdivide our smallest unit of length, the millimetre, we use an instrument called a vernier.

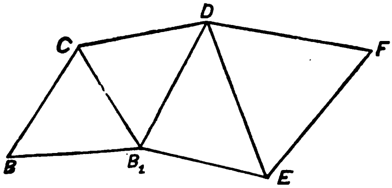

FIG. 12.

Suppose we should slip alongside a millimetre-scale a piece of paper upon which a space equal to nine millimetres has been divided into ten equal parts. It will be seen that each one of the divisions on the paper falls short of a millimetre by one tenth. We therefore look along the vernier until we find some line of its graduation which is exactly opposite a line on the millimetre-scale. The number of divisions on the vernier-scale up to this line will express the number of tenths of a millimetre, in addition to the whole number of millimetres. Thus, in Fig. 13, the reading is five centimetres, two millimetres, and five tenths of a millimetre, or .0525 of a metre.

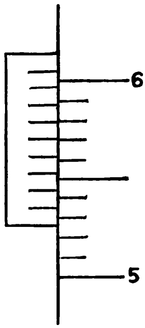

FIG. 13.

EXPERIMENT 37.—Make a vernier. Take nine millimetres on a fixed scale, and divide the space of nine millimetres into ten parts, by means of a scale* of proportional parts. It is evident that one of these ten divisions will fall short of a millimetre

* Such as is used by engineers and surveyors.

by one tenth of a millimetre. Attach this vernier (Fig. 14) to a piece of wood which can slip beside the metre-scale. Measure the diameter of a lead ball, by placing a piece of wood perpendicular to the metre-scale — allowing the ball to be inclosed by this piece of wood, the metre-scale, and the vernier, which lies beside the scale, and opposite to the piece of wood. (See figure.)

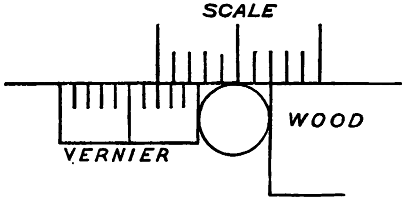

Fig. 14.

Experiment 38.—Find the average thickness of a piece of wire, by measuring it with a vernier-gauge in six different places, and taking the average of these measurements. Compare this result by that obtained as follows: Obtain the loss of weight of the wire in water, this gives the volume of the wire. The volume

$$= \text{area of cross-section} \times \text{length}$$
$$= \pi r^2 \times l = vol$$
$$r^2 = \frac{vol}{\pi l}$$
$$\text{or } r = \sqrt{\frac{vol}{\pi l}}$$

It is evident that, if we knew the pitch of a screw—or, in other words, the distance between the threads—and if this pitch or distance were uniform, we could mount a screw in a long nut, and interpose the object whose dimensions we wish to measure between the end of the screw and a fixed

support; and then ascertain how many whole turns and fractions of a turn of the screw are necessary to make the end of the screw pass over the space occupied by the body. Unless the screw is made with great care, the pitch is not uniform. With care in selection, however, one can be obtained, from a hardware-store or a mechanician, the pitch of which is sufficiently uniform through short distances.

EXPERIMENT 39.—Obtain a screw with a comparatively fine thread. Solder upon the head of the screw (*P*, Fig. 15) a circular tin plate, and glue to

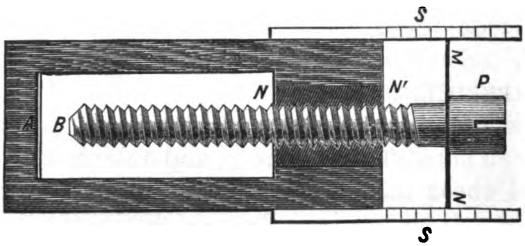

FIG. 15.

this a paper or horn protractor. Pass the screw through a metallic nut, and imbed this nut in one end of a suitable box or frame. Affix to the frame two scales, *S S*, in order to determine how many whole turns the screw makes. The fractions of a turn are given by the protractor or divided head, *M N*.

It is necessary to place a piece of brass against the end, *A*. The object to be measured is inclosed between *A* and *B*, the end of the screw. This apparatus can be converted into a dividing engine, by fixing the screw between *A* and *C* (Fig. 16), so that it can not move forward or backward, and can

only turn. In this case the nut *N N'* is allowed to move on the screw along a suitable groove, and a collar at *C* confines the screw.

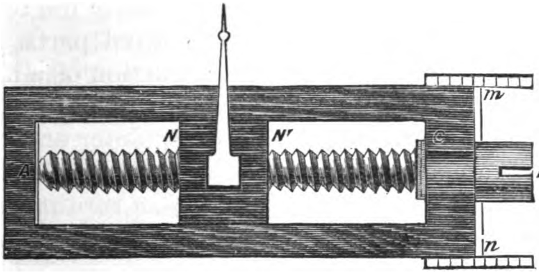

FIG. 16.

A little sharpened pointer attached to the movable nut will enable one to graduate scales or glass tubes. If divisions are to be made on glass, the glass is first covered with a thin coating of wax or paraffine. After the divisions are made, the glass tube can be immersed for a moment in hydrofluoric acid. In order to do this, take a piece of lead pipe of a suitable length and diameter. Close one end by hammering. Pour into the vessel thus made a sufficient amount of hydrofluoric acid (commonly called white acid by etchers on glass). A few preliminary trials with a roughly graduated piece of glass will enable one to determine how long the carefully divided glass tube should be immersed, in order that the divisions, and their appropriate figures or designations, drawn through the paraffine, may be etched. Two or three minutes generally will be sufficient.

EXPERIMENT 40.—Graduate a piece of paper into millimetres, and also into $\frac{1}{10}$ of an inch.

Suppose that the pitch of the screw of your micrometer, or dividing engine, is $\frac{1}{10}$ of an inch. It is evident that the head of the screw should make a complete revolution in order to move your pointer through $\frac{1}{10}$ of an inch. Since the head of the screw is generally divided into one hundred parts, you must reduce $\frac{1}{10}$ of an inch to a fraction of a millimetre, and calculate what fraction of a turn the screw-head will give. Move the pointer or pencil of the graduating apparatus over a millimetre. If the screw-head is divided into 360°, a turn through one degree will evidently move the screw through $\frac{1}{10} \times \frac{1}{360} = \frac{1}{3600}$ of an inch.

EXPERIMENT 41.—A cheap spherometer can be made in the following manner: File one end of a screw which has a fine thread to a point. Solder a round, metallic disk to the other end. Paste upon this disk a paper protractor. Sink the nut of the screw into a wooden base. Upon this base erect two standards 180° apart. Graduate these standards in whole turns of the screw. Provide the base with three metallic legs which are pointed. Three long brass screws, pointed at the end, will answer for these legs. In turning the head of the screw, and therefore the protractor, the half circle of the protractor should always just glide by one of the standards, and the reading of the whole turns, and the fractions of a turn, can therefore be taken at one of the standards. In sinking the screw-nut into the wooden base, a hole should, of course, be made through the wood below the nut, so as to allow the screw to pass through the base. The spherometer is then placed upon a piece of plane glass, and the screw turned until it just touches the plane glass.

This will be when the reflection in the glass appears to touch the screw. This is the zero of the scale on the head. The instrument is then placed upon a curved surface, such as a lens, and the screw elevated until it just touches the curved surface. The number of divisions on the head which pass by the fixed standard, or index, gives the difference in height between that of the plane surface and that of the curved one.

Obtain in this way the thickness of plates of mica, and compare the values with those obtained by a vernier-gauge. (See Appendix.)

EXPERIMENT 42.—Obtain the specific gravity of an iron cylinder by measuring its diameter and its length and weighing it. The measures of length give its volume, and therefore the weight of the water which it would displace. Care should be taken that the cylinder should always be inclosed by the arms of the gauge at the same place. If the ends of the cylinder are concave, a small piece of sheet-brass or platinum can be placed in the ends of the cylinder, and from the length obtained the width of the brass can be subtracted. A spherometer can be used to make these measurements.

With an accurate vernier-gauge we can measure very small subdivisions of a millimetre. Another device for accomplishing the same purpose is called a micrometer. It consists of a screw, the head of which is enlarged into a circle, which is divided generally into one hundred parts. If the pitch of the screw—that is, the distance between any two threads—is one millimetre, it is evident that a portion of a turn can be read in hundredths by observing how many of the divisions of the head of the

screw pass a fixed point outside the screw. A micrometer can be supported in various ways. If it is used to measure the curvature of a surface, it is called a spherometer.

Micrometers are generally attached to the eye-pieces of microscopes and telescopes. In this case they are used to push a fine cross-hair across the field of view, and thus measure the diameter of a small object, or small distances between two objects. Thus, let *O* (Fig. 17) be a fixed circle or aperture at the focus of the eye-piece of a microscope or telescope, and let the screw *P* move a frame, *A*, *B*, *C*, *D*, *E*, *F*, provided with a cross-hair, *B E*, across this aperture. The springs *n n* serve to push the frame against the screw. The distance that the movable hair *B E* passes over will be given in fractions of the pitch of the screw by the head of the screw *M N*. In this way astronomers can measure the diameter of a small planet.

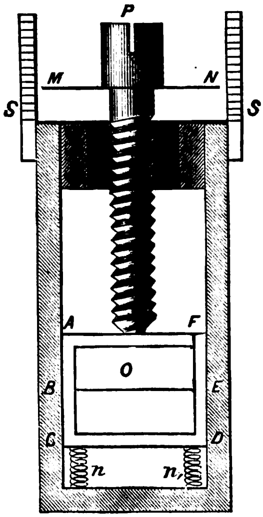

FIG. 17.

Such a micrometer is also necessary in measuring the expansion of a bar of metal under the effect of heat. This is too small to observe with the naked eye at ordinary temperatures, and hence microscopes provided with micrometers in the eye-piece have to be used. The small expansion of metals when heated becomes a very important considera-

tion when a base-line is to be measured or a piston-rod is to be fitted into the cylinder of a steam-engine. A bar of brass one foot in length increases in length only 0.0000187 of itself for one degree rise in temperature, yet the force exerted is enormous, and the expansion must be carefully estimated.

The microscopes and verniers we have described can be attached to any instrument with which we wish to measure the dimensions of small objects.

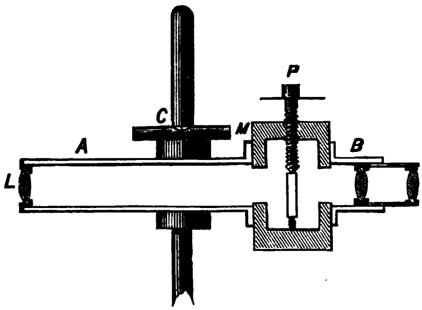

FIG. 18.

In order to attach such a micrometer to a telescope, obtain a brass or tin tube (A, Fig. 18), place a small lens of about twelve inches focus (see Appendix) in a piece of cork, and slip it into the end L of this tube. Screw the other end of the tube upon one side of the small box containing the micrometer M. Screw another piece of tin tubing, b, upon the other side of the micrometer-box. Into this last tube slip an eye-piece containing two small lenses. The interior of the tin tubes and the micrometer-box should be blackened. The telescope should be provided with a little spirit-level at C. The telescope can be made to slip up and down a vertical rod, similar to that which is used by photographers for a head-rest. Such an instrument is useful in measuring, for instance, the

height of a mercury-column. In order to do this a metre-scale is suspended vertically by means of a plumb-line beside the column to be measured, and the whole divisions are read from this scale, and the small interval between the last division on the scale and the top of the mercury-column is read by means of the micrometer.

The cross-hair should be a fiber of silk, obtained by untwisting a piece of silk thread, and taking the ultimate fiber. This cross-hair should be at the common focus of the lens at *L*, and the eye-piece. The metre-rod (Fig. 19), suspended in a vertical position by means of a plumb-line, rests against a vertical wire, *E F*, of which one end, *F*, just touches the surface of liquid in the reservoir *R*. Knowing the length of *E F*, we can add it to the indications of the metre-rod.*

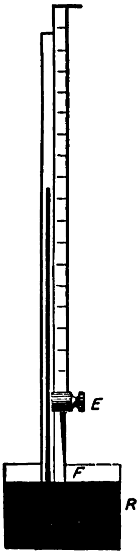

FIG. 19.

EXPERIMENT 43.—Ascertain whether an India-rubber strip is perfectly elastic. A substance is perfectly elastic when it recovers its original form after any distortion. Measure a certain length, stretch it a certain distance, and then measure again, and so on, increasing the amount of stretching. From your observations deduce the limit of elasticity. An ordinary scale will answer for this experiment.

EXPERIMENT 44.—When a substance is stretched

* It is well for the student to construct a micrometer such as I have described, for the practical knowledge that is thus obtained in optics.

beyond a certain limit, the amount of stretching does not increase with the weight applied. Up to this point, however, two grammes produce twice as much elongation as one gramme, three grammes three times as much as one gramme, and so on. In other words, up to a certain limit the ratio between the increase in length and the weight is constant. Find this limit for India-rubber.

EXPERIMENT 45.—We compare the elasticity of different substances by observing the elongations which a weight of one gramme will produce in a cylindrical or prismatic bar of the substance one centimetre long and one square centimetre in section, $\frac{1}{\text{elongation}}$ expresses the difficulty of stretching. By measuring the elongation within the limits of elasticity of a rod or cord of suitable length and section, we can readily find by proportion how much a rod or cord one centimetre long and one square centimetre in section will be elongated. The co-efficient of elasticity is the ratio of increase of a bar of unit section when it is stretched with the unit force to the entire length of the bar. When the elongation of a metal wire is observed, it is best to pass it through the center of a thin brass tube, which is clamped to the wire by a screw (*S*, Fig. 20). The length of this brass tube remains unaffected by the stretching weight *W*, and the yielding of the support *M* will be obviated if we measure from the end *b* of the brass tube to a mark, *a*, on the wire. Measurements with metal wires require a microscope or telescope to read the small elongation.

EXPERIMENT 46.—Find the internal diameter of a given glass tube.

The volume of a cylinder is the area of its base multiplied by its altitude. We have seen that the area of a circle is expressed by πr^2, in which π is 3.1416, and r is the radius of the circle. Close the tube at one end by a cork, and fill the tube with water. Weigh this water, and the number of grammes will give the volume of the cylinder in cubic centimetres; from this volume obtain the radius of the cylinder. Repeat this experiment with mercury.

FIG. 20.

If the internal diameter is small, draw into it a thread of mercury, two or three inches long; measure its length at different points, moving it along by inclining the tube. Knowing the weight of the mercury, by placing the glass tube in a balance, and subtracting the weight of the glass tube from that of the glass tube with the thread of mercury, we can obtain the radius of the bore of the tube from the expression $W = l\pi r^2 s$, in which W is the weight of the mercury, l the length of the mercury-column, π is 3.1416, r the radius of the tube, and s the specific gravity of mercury. This operation of determining the internal radius of a tube at different parts is called calibration, and is necessary whenever we wish to graduate the scale of a thermometer; for, if the bore is not uniform, it is evident that the divisions on the thermometer-scales must be unequal. With the instruments we have described we can now ascertain the variations of bodies under the influence of heat or of any force of distortion. The measurement of these variations is extremely important, for

upon these measurements is based the great doctrine of work, which expresses the law that the force multiplied by the space passed through gives the work which is done. This will be explained later. All our experiments are directed toward obtaining a clear idea of the doctrine of work, and it is necessary, in the first place, by accurate measurement, to obtain an idea of the change of dimensions of bodies under the effect of weight or of pressure.

We have said that our operation of weighing is really a determination of mass. The mass is the volume of a body in cubic centimetres multiplied by the density of the substance, and hence we have

$$M = Vd.$$

Now, the weight of a body is the mass multiplied by the pull of gravitation upon every particle, hence the weight is the mass multiplied by the force of gravitation, or

$$W = Mg. \qquad \text{Eq. (1),}$$

where g is the force or pull of gravitation on every unit of mass. If, then, the weight of a body is given, we can always determine the mass for $M = \frac{W}{g}$ from Equation 1.

CHAPTER V.

MOTION.

We have devoted considerable space to measurement of space of volume and of mass; for the ideas we first gain in childhood are ideas of distances, of space more or less occupied with matter. To the child the earth appears to be standing still in the heavens, and the conception that all bodies are in motion, although they may seem to be at rest, is one of maturity. We perceive after a time that there is a science which treats of the relative position and motion of bodies, and of the regular succession of events in Nature. Day follows night. The temperature at noonday changes with the season of the year. The height of the tide alters with the relative position of the sun and moon. The frost-figures on the window-pane vary with the amount of moisture in the air and the condition of the window-pane. Physical science has been defined as that department of knowledge which relates to the order of Nature, or, in other words, to the regular succession of events.

In order to fix our ideas, we speak of a material particle. This is a body so small that we can neglect the distances between its various parts. When we investigate the law of attraction between

a small stone and the earth, we consider the stone as a material particle, for we are not able to take into consideration the irregularity of the stone, and the different effect of its various parts. We are obliged to assume that there are molecules or ultimate particles of every substance, in order to fix our ideas and to simplify our calculations; but, when we consider the rotation of a particle about some axis, we can not neglect the different effect of its various parts: for the effect of the rotation of a part far from the axis differs greatly from that of one close to the axis, the farther particles moving with a greater velocity.

The universe is so extended and so complicated that it is necessary for us to restrict our measurements to a certain area, or what is called a configuration. In order to get a clear idea of a State like Ohio, we must have a map of it, in which certain parts are connected with others by lines which have been extended from some base-line. In order to obtain a conception of a complicated piece of machinery, we must have a diagram of the motion of its various parts. In order to study motion we must carefully examine our thoughts in regard to space and time. For the purpose of physical science, the object of which is to investigate successive events, these ideas must be definite. Our ideas of space and time are relative, and our ideas of motion are relative, since they are based upon ideas of space and time.

If we should at any instant desire to fix the position of three points on three circles of card-board, which are revolving near each other with the same velocity, with reference to any outside point, a dia-

gram would be necessary. Knowing the rate of revolution of the circles, we could construct on a diagram the position of the points with reference to an outside point at any subsequent time. We could do the same if the circles revolve with different velocities; we could apply the same method of observation to the motion of the parts of a machine. From the results of our observation of different configurations, or diagrams, we could write a treatise on mechanics; such treatises are called works on kinematics. Some knowledge of geometry and trigonometry would be necessary to simplify our observations, but we should not have to study any change of position of the circles or the machines, or to investigate what causes the movement of the various parts. When we take into account the mutual action between bodies, the science of motion is termed kinetics; and when we investigate force, we embody our results under the head of dynamics.

We shall find that our observations in dynamics resemble those of a business-man who (see Maxwell's "Matter and Motion," page 54) opens an account with another business-man. The same transaction is called by one man buying and by the other selling; and it is called trade when we take both parties into consideration. These transactions are entered by the two merchants on opposite sides of their respective ledgers; and the accountant, in comparing the ledgers, "must remember in whose interest each book is made up." In dynamical inquiries we must remember also which of two bodies we are dealing with, and set down to the respective bodies the forces which belong to them. We shall see also

that our idea of force is gained by the rate of change of motion.

Before beginning our study of motion, let us ask ourselves what is the object of our study. We can not investigate the origin of force. We can not know what is the ultimate cause of anything—why the planets were set in revolution, and why there is a force of attraction which acts instantaneously throughout the universe. We must content ourselves in studying the changes and variations of motion. We say in general that a force or forces produce motion and a change in motion. We can only study these changes by noticing their rate of change, just as the political economist studies the movement of gold by the rate of exchange. The gold represents a certain amount of energy. We think of force through the manifestation of exchanges. The rate of the exchange may be high or low; and we therefore speak of the intensity of a force, and we get our conceptions from noticing the rate of change of motion. It is necessary in unscientific language to speak of forces; thus, we have the expressions, the forces of Nature, the force of magnetism, the force of electricity, the vital forces, etc. Modern physics no longer treats of different forces; it does not strive to ascertain what electricity is, but it endeavors to ascertain how the manifestations of the exchanges of energy in electrical phenomena are connected with the light and heat phenomena that result, and how these heat phenomena are related to the work done by a stone falling under the influence of gravitation. If we knew what electricity is, we should know what the ultimate cause of motion is. All that we can expect

to discover is the relation between the exchanges of energy which are manifested by electrical phenomena, and the exchanges in heat and in motion in general.

A treatise on mechanics could be written in which the law of attraction of gravitation might not be mentioned. The law of attraction of magnetism might take its place; and we could prove the law of the parallelogram of forces, the law of couples, and, indeed, study every change of configuration of a system submitted to magnetic force, just as we commonly study dynamics by the aid of the law of gravitation. The effects of the attraction of gravitation, however, can be more universally observed, and the measurements can be more conveniently made.

Up to this point we have not considered any problems which arise when the motion of bodies is considered. When we reflect that no particle on the earth can be said to be at rest; that we are whirling through space; that what is above us by day is below us by night—we see the need of fixing our ideas in regard to motion in general, and the necessity of having some standard which will enable us to estimate the motion of all bodies, just as we investigate the relative masses of different bodies by reference to a fixed standard.

The laws of motion are three, and were first clearly enunciated by Sir Isaac Newton. They are as follow:

LAW I.—Every body perseveres in its state of rest or of moving uniformly in a straight line, except in so far as it is made to change that state by external forces.

LAW II.—Change of motion is proportional to the impressed forces, and takes place in the direction in which the force is impressed; or, as it is sometimes expressed—

The change of momentum of a body is numerically equal to the impulse which produces it, and is in the same direction.

LAW III.—Reaction is always equal and opposite to action—that is to say, the actions of two bodies upon each other are always equal and in opposite directions.

Let us begin our study of motion by investigating the fall of a stone, or any body, to the earth. The stone falls toward the earth because there is an attraction between it and the earth. It starts from a state of rest and acquires a certain velocity at the end of a certain time. If it rolls down an inclined plane, it moves with a certain velocity in a horizontal direction, and with another velocity in a vertical direction. If a body is shot up an inclined plane against the force of gravitation, it will move a certain space in a horizontal direction and a certain space in a vertical direction. To reach the top of the plane a person can either walk up the plane, or walk a distance in a horizontal direction equal to the base of the inclined plane, and then climb up a ladder equal to the vertical height of the inclined plane. Suppose two persons should start from the same point, one desirous of reaching the vertex of an inclined plane at the same time that the other reaches the foot of the vertical line of the inclined plane, it is evident that the man $m_{\prime}$ (Fig. 21), running along the base, should always be directly under the man, m, running up the plane. If we represent the

velocity with which *m* is running up the plane by the length of the line *m a*, the velocity which the man *m*, must have will be *m b*, which is found by dropping a perpendicular from the extremity of the line representing *m*'s velocity to a horizontal line, *m b*. In other words, we project the actual velocity by perpendiculars upon the line along which we wish to obtain the relative velocity. In the same manner, if we wish to find how fast a man must climb the vertical height *B C* of the plane in order to reach the height *B* at the same time that another man, *m*, ascends the hypothenuse of the plane, we project the velocity *m a* by perpendiculars upon the height *B C*, and the line *a b* would represent the corresponding velocity that the man ascending the plane must have, at any time.

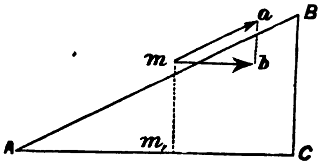

FIG. 21.

In order to denote the rates of movement along the height and base of an inclined plane in terms of the rate along the hypothenuse, we must adopt some convention which will abbreviate such an account as we have just given.

In a circle with a radius of 1 (Fig. 22), let us draw any angle, *a*. In the small inclined plane *A b c*, we have taken the hypothenuse *A b* as 1. Let us call for convenience the base *A c*, the cosine of *a*, generally written (cos *a*), and the alti-

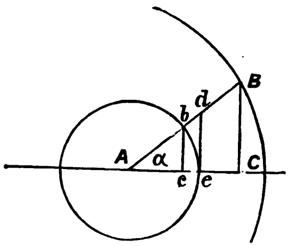

FIG. 22.

tude $b\,c$, the sine of a (sin a). These are merely convenient designations for the sides of an inclined plane. Let us also draw a tangent to the circle at e, and call the line $d\,e$ tangent of a (tang. a). By actual measurement we shall find that $\frac{d\,e}{A\,e} = \frac{b\,c}{A\,c}$, or (since we have taken the radius of the circle = 1, or $A\,e = 1$), $\frac{d\,e}{1} = \frac{\sin a}{\cos a} = \tan.\ a$. Also we find by actual measurement that $\frac{A\,c}{A\,b} = \frac{A\ C}{A\ B}$, or (since $A\,b = 1$), $A\,c = \cos a = \frac{A\ C}{A\ B}$, or $A\ C = A\ B \cos a$. Also $\frac{b\,c}{A\,b} = \frac{B\ C}{A\ B}$, or $b\,c = \sin\ a = \frac{B\ C}{A\ B}$, or $B\ C = A\ B \sin a$. Hence we can always find the projection of a line which represents a force or a velocity, upon another line, by multiplying the line representing the force or velocity by the cosine or the sine of the angle between the two lines. The values of the conventional sine and cosine, or, in other words, the base and height of inclined planes having acute angles at the center of a circle whose radius is one, have been calculated and tables formed (see Appendix).

The mathematical conceptions which we employ to render our thoughts more concise when we investigate motion serve, therefore, to express the velocity or the force along any line which is inclined to the direction of the motion.

For the purposes of this treatise we need only to comprehend the signification of sine and cosine and tangent of an angle, and this comprehension can be

obtained without any previous knowledge of mathematics save that of elementary algebra and the elementary ideas of proportion.

EXPERIMENT 47.—Draw an inclined plane, measure the angle at the base by means of a protractor, and taking the values of the cosine and the sine from a table of natural cosines and sines, multiply the length of the hypothenuse by these values respectively, and show that we thus obtain the base and the altitude of the inclined plane.

EXPERIMENT 48.—In order to obtain an idea of the relations of our conventions in regard to angles and the sides of a triangle, draw a circle with a radius of one foot; lay off various angles at the center of the circle. One side of each of these angles being a horizontal diameter of the circle, draw the sine and cosine for each of these angles. The cosine is by definition equal to $\frac{A\,c}{A\,b} = \frac{A\,c}{1}$, and the sine equal to $\frac{b\,c}{A\,b} = \frac{b\,c}{1}$. Measure $A\,c$ and $b\,c$ for the different angles, and find their ratio to the radius of your circle, or to 1, and compare the results you obtain with those corresponding to the same angles in a table of natural sines and cosines. Do the same for the tangents.

In our measurement of masses in obtaining specific gravity, we have supposed that the attraction of gravitation is the same upon the weights as upon the body to be weighed, and thus instead of weights we have compared masses, for our weights are only correct for the latitude of Paris. We can also measure equal masses in another way without any reference to the force of gravitation. Suppose we should

attach a certain mass to an India-rubber cord, swing it around in a horizontal circle, and observe the elongations of the rubber cord. Our experiments on elasticity show us that the same force elongates the rubber to the same amount as long as we keep within certain limits of elasticity. If we swing the two bodies around with the same length of cord and with the same velocity, and find that the elongation of the rubber cord which serves as a radius of the horizontal circle is the same for both circles, we know that the bodies have the same mass. We have substituted in this experiment the tension along the cord for the pull of gravitation. If it were not for this tension, or, as it is sometimes called, centripetal force, and if it were not for the force of gravitation and the resistance of the air, the mass would continue to move in a horizontal plane, for the second law of motion states that "change of motion is proportional to the impressed force, and takes place in the direction in which the force is impressed." Newton understood by the word motion what we call momentum, that is, the quantity of matter moved, multiplied by its velocity, and the term "impressed force" is what is understood by the word impulse, "in which" (Maxwell's "Matter and Motion," p. 62) "the time during which a force acts is taken into account as well as the intensity of the force."

In dynamics, we say that two bodies have equal mass when equal forces applied to them will produce equal changes of velocity in equal times.

The unit of mass in England and America is 1 pound; one seven-thousandth part of a pound avoirdupois is called a grain. The French stand-

ard of mass is called a kilogramme; it is equal to 15,432.34874 grains, or 2.2 pounds.

When a mass is attracted toward the earth, we notice that it moves faster and faster; it strikes with a certain blow, or, as we say in common language, it strikes with a certain force. The greater the distance through which it falls, the greater the force of its blow. When we lift a weight, we feel that we are overcoming a certain force. We could compare the muscular force with which a person strives to push us, with the pull that we must give to lift a given mass against the force of gravitation. In a certain sense a weight can be taken as a force; it is the evidence of a pull or force. We speak sometimes of a force of forty pounds, thus comparing weight to force. This method of estimating force is termed gravitation measure. Unless we measure the force of gravitation at the place where we estimate force, our results can not be compared with those obtained at any other place on the surface of the globe, for the pull of gravitation varies with our position on the globe.

We therefore take as a unit of force that force which acting on the unit of mass will give it a unit velocity in a unit of time. The force of gravitation acting on a gramme will give it a velocity of 980 centimetres (in the latitude of Paris) at the end of one second; hence the weight of one gramme can be represented by 980 when the unit of weight is the gramme, the unit of distance the centimetre, and the unit of time is one second.

In the French system, the force which will give to one gramme a velocity of one centimetre per second is called a dyne.

Let us now investigate the motion which results from various impulses. Motion is of two kinds: uniform motion and accelerated motion.

The motion of the second-hand of a watch is uniform. It is evident that, if it moved faster one minute than it did during another minute, the minutes would not be of the same length, and therefore we could not estimate time. A second-hand makes sixty ticks in a minute. The second-hand passes round the circumference of a circle once in a minute. If we divide the circumference of a circle into sixty parts, the second-hand will pass over sixty of these parts in a minute. It takes one second to pass over one of these divisions. If the length of one of these divisions were one inch, it would pass over one inch in one second. We can estimate its speed, therefore, and say that it is one inch per second. It will pass over just one inch in a second at any point on the circumference of the circle. Suppose that it passes over two inches in a second, its velocity will then be two inches per second. We estimate the speed or velocity by the space the moving point passes over in one second.

What would be the space passed over by the end of a second-hand of a clock in one minute if the space passed over in one second is three inches? This would be three inches, or the space passed over in one second—which is the velocity—multiplied by the number of seconds in one minute, which is sixty. That is, the whole space passed over would be $3 \times 60 = 180$ inches. We express this by saying that in uniform motion space = velocity multiplied by the time in seconds, or $s = vt$.

What would be the space passed over by the

end of a second-hand of a clock in one hour, if it passes over one tenth of an inch in one second?

If the second-hand of a clock went faster and faster, we could have no fixed standard of time, unless we compared our clock with some clock which never varied from a uniform rate. Suppose that the second-hand of my clock passed over one inch the first second, two inches the second second, three inches the third second, and so on. This would be a case of what is called accelerated motion. The second-hand is going faster and faster, and it would evidently be very difficult to tell the time by such a clock. The second-hand would gain fast upon the second-hand of the standard clock which is marking the time uniformly. We could, however, calculate the time from our clock, whose second-hand is going faster and faster, by comparison with that of our standard uniform clock.

Most of the examples of motion which attract our attention are not uniform. The motion of the earth around its axis, giving us night and day, is uniform; and this motion affords us our common standard of uniform time. It is very difficult, however, to obtain uniform motion. This is attained by great care and skill in the manufacture of watches and clocks. It is found very difficult to make an engine of any kind run uniformly. The human heart, which is a species of engine, beats from sixty to eighty times a minute; at times it beats very uniformly, so that we could estimate the time of a boat-race by it, but our eagerness and excitement would speedily make its beats increase, so that it would cease to be a standard of uniform time. The kinds of motion which we see in every-day life are

not uniform; indeed, if there were many cases of uniform motion we should not have to pay so much for our watches and clocks.

The commonest case of accelerated motion is that of a falling stone. When we let a body fall, its velocity steadily increases as the time increases. We say that it is attracted to the center of the earth by the force of gravitation. Place a small magnet, made from a piece of watch-spring, on a bit of cork, and float it on the surface of water; bring a strong magnet within a short distance. The small magnet will be attracted toward the large and stronger magnet, and will move with accelerated motion.

Suspend a light pith-ball, or a light feather, by a string, and, having rubbed a piece of sealing-wax with silk, bring the sealing-wax into the neighborhood of the pith-ball; it will move toward the sealing-wax with accelerated motion.

It is necessary to obtain clear ideas in regard to the accelerated motion produced by these various manifestations of attracting force. We shall see that work is measured by the force acting through a certain space. Since our spaces are passed through, in general, not at a uniform rate, the importance of being able to estimate the space passed through in a given time is very great. Before showing in a rigorous manner that work done in any case is proportional to the space through which the force acts, it is only necessary to ask whether you would prefer to lift a heavy bucket from a well sixty feet in depth, turning the windlass faster and faster each second, or from a depth of forty feet in the same manner; or if you would prefer to lift the

same bucket from those depths, turning the windlass at a uniform rate of speed. It is evident that the effort would be different in each case, and that we must have some standard in order to compare the different amounts of work we do in the different methods of working. We shall find that the work done in lifting a weight is the same whether we take a day or a year to lift it a certain height. The total amount of work, however, we do in a given time depends upon the number of times we lift the weight.

Let us first try simple methods, and afterward proceed to more accurate and scientific methods of experiment.

EXPERIMENT 49.—*To ascertain the Laws of Falling Bodies.*—*A* and *B* are two grooved wheels placed upon a board and connected by a string. When *A* is turned by a crank *C*, the smaller wheel *B* revolves more rapidly than the wheel *A*. If the circumference of the wheel *A* is twenty-four inches and that of *B* three inches, and if *A* revolves once in one second, it is evident that *B* will in one second measure off its circumference as many times as three inches is contained in twenty-four inches, or eight times.

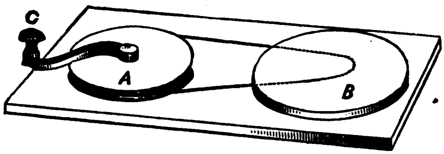

FIG. 23.

The small wheel *B* will therefore revolve eight times in one second. Let us now place a large, flat piece of wood upon the axle of *B*. This flat circle

of wood will also go round eight times in one second, or will make one revolution in $\frac{1}{8}$ of a second. Let us cover this flat circle with a thin layer of moist clay or dough rolled out to a suitable consistency. If we could make the wheel *B* revolve uniformly eight times a second, or once in $\frac{1}{8}$ of a second, we could let two small bullets or lead shots fall from heights separated by a measured interval, and measure the distance of the dints of their blows in the clay or dough from each other. If the lead balls were suspended vertically, one of them just above the other, at a distance of one foot, and if they were released at the same instant, the wheel would revolve through a considerable angle after the first ball had struck the clay and before the second ball could reach it. Suppose this angle is measured, and is found to be $\frac{1}{4}$ of the four right angles contained about the center of *B*, or, in other words, one right angle or $\frac{1}{4}$ of a whole turn, we should know that the interval between the time of striking of the first ball and that of the second is $\frac{1}{4}$ of $\frac{1}{8}$ of a second (since the wheel turns entirely around in $\frac{1}{8}$ of a second), or $\frac{1}{32}$ of a second. Suppose instead of two lead balls we have but one, we could not tell the time it has taken to fall through a measured height unless we could record on the wheel the exact instant we released the ball: for we must measure from some starting-point on the wheel.

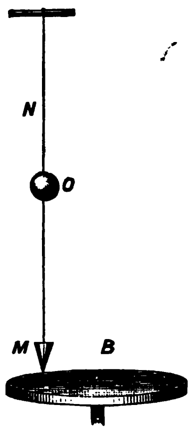

Fig. 24.

Let us, however, make an approximation in this ex-

periment and afterward see how much this approximation is—in other words, how much of an error we shall make. Suspend a lead ball over the flat circle *B* (Fig. 24), a convenient distance out from its center. Provide the ball with a light string to which a weight, *M*, is attached. The weight *M* is pointed and remains at a very small distance from the surface of the clay or dough upon the wheel *B*. Set the wheel *B* in rotation, and, when this rotation has become uniform, burn at *N* the string which sustains the ball *O*. The pointed weight *M* will strike the clay approximately at the instant of burning the thread, and the time that it takes the ball *O* to fall through the distance *O M* will be measured by the part that the angle subtended between the mark made by *M* and the mark made by *O* in the clay on the circle *B* is of the whole angle about *B*, or four right angles.

In order to maintain the revolution of *B* uniform, suspend a heavy weight at the end of a strong string of a suitable length, perhaps one metre or three feet, and make it revolve around a circle drawn on the floor, the center of this circle being directly beneath the heavy weight when it is at rest. This apparatus is called a conical pendulum. You will find by timing its revolutions with your watch that it executes a complete revolution always in the same time, although of course the weight does not keep always on the same circle. Having ascertained the time of the pendulum, fol-

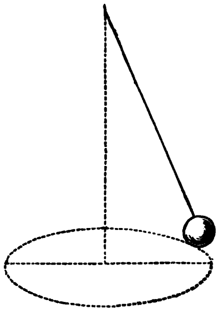

FIG. 25.

low its motions with the crank arm attached to the circle *A*. A little practice will enable you to maintain the circle *A* in uniform rotation with the conical pendulum. The circle *B* can be made to run at any suitable rate by increasing or diminishing its size, and also that of *A*.

Now let an assistant drop the ball *O* from different heights, and tabulate the results thus:

Spaces or heights.	Time of falling.	Square of time of falling.

It will be found that the heights will be to each other as the squares of the times, or $\frac{h}{h_{,}} = \frac{t^2}{t_{,}^2}$.

In any one case, therefore, the height must be proportional to the squares of the times it has taken to fall through this height. Hence $h = Ct^2$, in which C is a number we must multiply by the square of the time in order to obtain the height. It is evident that the time squared must always be multiplied by the same number, otherwise the heights would not be proportional to the squares of the times alone, but also to this number if it varied. Since it is a constant multiplier, it cancels out when we obtain the ratio between any two heights through which the weight falls. By trial, taking the height fallen through in feet and the time in seconds, we find that this number by which we must multiply the square of the time in order to obtain the height, is sixteen feet (about 16.08 for New York).

We can therefore write—

Space fallen through = 16 multiplied by time squared, or $s = 16t^2$.

Hence, if we make $t = 1$, or one second, we find that a body will fall 16 feet in the first second. If we make $t = 2$ seconds, the space fallen through will be 64 feet. When $t = 3$, the space will be 144 feet, and so on.

The principal source of error in this experiment arises from the difficulty of maintaining the rotation of *A* uniform. A little practice, however, will give fairly uniform rotation.

Another source of error is in neglecting the slight distance of the pointed weight *M* from the surface of the clay. This distance, however, can be made very small in comparison with the distance through which the ball *O* falls.

EXPERIMENT 50.—To obtain the final velocity of a falling body.

Suspend two balls, at different distances apart, by passing the connecting thread over a wire support, as in Fig. 26. Place the revolving circle *B* with its level surface of clay or dough beneath, so that the balls *F* and *E* snall lie on the same diameter of *B* (as in figure). Set *B* in uniform rotation, and then let an assistant burn the string at *n* with a match. The velocity attained by *E* during the last few inches of its fall will be approximately that

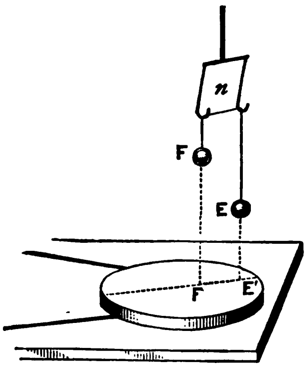

FIG. 26.

of *F*. If *F* is sufficiently near to *E*, this velocity will be measured by the relative positions of the blows of *F* and *E* on the clay of the circle *B* (Fig. 26).

The part that the angle between the diameters upon which *E* and *F* have fallen is of the whole angle, or 4 *RA*, will give the velocity of *E* and *F* during the last inch or inches of its fall. By arranging the weight *E* so that it may fall through sixteen feet, and placing the weight *F* slightly above it, we shall find that the falling bodies are moving at the rate of thirty-two feet per second. In one second, therefore, a falling body under the influence of gravitation will acquire a velocity of thirty-two feet per second.

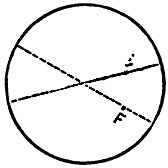

Fig. 27.

CHAPTER VI.

PENDULUM MOVEMENTS AND THE MEASURE OF FORCE.

PENDULUM movements may be said to constitute our chief study when we investigate the laws of motion in the subjects of sound and light. The pendulum also plays an important part in our investigation of the velocity of moving bodies. The simple gravitation-pendulum is a weight suspended at the end of a string or rod. A weight suspended by a slightly twisted wire or rod and the magnetic compass constitute horizontal pendulums. An elastic rod set into transverse vibration can be used as a rapidly moving pendulum. A tuning-fork is used when we wish to employ a very rapidly moving pendulum. The shortest gravitation-pendulum which we could use conveniently would not swing faster than ten times a second; and, therefore, by its use we could only measure $\frac{1}{10}$ of a second. A tuning-fork can be made which will execute many hundred swings in one second. To show that the motion of a tuning-fork and that of an ordinary gravity-pendulum are similar, proceed as follows:

EXPERIMENT 51.—Suspend a little tin can by a string or strings about one metre long. Make a small hole in the bottom of the can; fill the can with fine sand. Set the pendulum to swinging, and

draw a board or stiff piece of cardboard beneath the pendulum-bob, at right angles to its direction of swing. In this way we shall get a sinuous line on the board, formed by the sand. Then mount a large tuning-fork horizontally in a vise. Provide one of its prongs with a small bit of wire, which shall just touch a plate of smoked glass. Set the fork into vibration by striking its prongs with a wooden mallet, and quickly draw the smoked glass beneath the prong. You will obtain a sinuous line which is similar to that obtained by means of the gravity-pendulum.

EXPERIMENT 52.—*Torsion-Pendulum.*—Suspend a wire vertically from a vise screwed to a firm support—a horizontal rafter or beam. Attach the lower end firmly to a cylindrical vessel; a tin pail will answer. Fill this pail with lead bullets. Paste a slip of paper, which has been graduated into millimetres or tenths of an inch, around the rim of the pail, and provide two vertical wires which will serve to direct your eye always in the same vertical plane. Twist the suspending wire, and you will perceive the graduations on the rim of the pail pass these vertical wires to and fro. Observe how many of these graduations pass the wires while a clock is beating seconds, or while your seconds-pendulum is beating seconds. You will in this way obtain the value of one graduation in fractions of a second. Then, when the seconds-clock beats a second, observe how far in graduations the zero-mark (which is opposite the vertical wires when the pendulum is at rest) is from the vertical wires. The time of crossing will then be one second, plus the fraction of a second indicated by these graduations.

The experiments we have performed thus far in motion give us only comparatively rough values of the acceleration of bodies under the influence of a stress, such as is exercised by gravitation. It is evident that a more exact method must be devised. The pendulum gives us an accurate method of estimating time, and, as will also be seen, of measuring force.

Before we can understand pendulum movements it is necessary to comprehend what is called the resolution of forces. Hitherto we have had to do only with the force of gravitation, and the motion we have studied has been in the direction of the pull of this force. Upon every particle of a body a vertical force acts which attracts the body to the earth; for the sum of the attraction upon each unit of mass, one force can be substituted which will pass through what is called the center of gravity of the body.

Experiment 53.—Find the center of gravity of an irregular sheet of card-board. Attach a small piece of lead to a thread, stick a pin through one corner of the card-board, making the hole so large that the card-board will readily move around the pin. Stick the pin thoroughly into a vertical support, so that the sheet of card-board will hang vertically. Suspend the little plumb-line you have made with the thread, and the bit of lead, from the pin, and draw a line on the card-board along the plumb-line. Suspend the card-board in the same way from another corner, and draw the new course of the plumb-line. Where these lines intersect will be the center of gravity of the card-board. Verify this by balancing the card-board on a vertical point.

EXPERIMENT 54.—Find in the same way the center of gravity of a triangular piece of card-board, and discover how far the center of gravity of any triangle is from its base and from its vertex.

EXPERIMENT 55.—Suspend a rod from its center of gravity by a string which passes over a little smooth pulley and is weighted at its end (Fig. 28). It will be found that the weight W, necessary to keep the system in equilibrium, is equal to the weight of the rod. Then place equal weights at the ends of the rod. The additional weights at the end of the string necessary to keep the system in equilibrium will be equal to the sum of the weights W_1 and W_2. The weights W_1 and W_2 represent equal forces acting vertically upon the ends of the rod. The pull on the suspending thread, therefore, is equivalent to the pull of these two vertical forces, together with the weight of the rod. Since the pulley is smooth, the tension on the suspending thread is everywhere the same, and is equal to the pull of the force due to the weight W. We see that the resultant of two equal parallel forces bisects the distance between these forces.

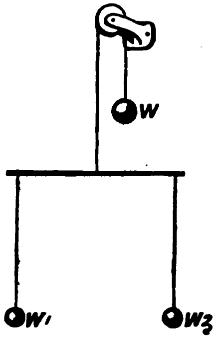

FIG. 28.

EXPERIMENT 56.—Repeat the preceding experiment, placing unequal weights at the ends of the rod. It will then be found necessary to move the point of attachment of the thread nearer the heavier weight. Ascertain the relation between the distances of this point of attachment from the points of attachment of the two weights. In this case

also it will be seen that the resultant of the two vertical forces is also a vertical force, and is equal to the sum of the two vertical forces.

EXPERIMENT 57.—Balance a light stiff rod horizontally upon a knife-edge, then attach equal weights to its two ends: the rod remains horizontal. The resultant passes through the knife-edge, for we can replace the pressure on the knife-edge by the pull on the string in Experiment 55. Place unequal weights at the end of the rod and it will be necessary to move the knife-edge or fulcrum nearer the greater weight. When the system is balanced on the knife-edge, it is evident that we can again replace the pressure on the fulcrum by the tension on the string in Experiment 56. The relation between the length of the lever-arms and the weights must therefore be the same as that obtained in Experiment 56.

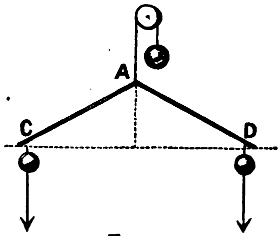

FIG. 29.

EXPERIMENT 58. — Replace the straight rod in Experiment 55 by two rods jointed firmly together at A (Fig. 29). If the rods are of equal length and of equal weight, the direction of the resultant will bisect the line joining C and D, and a straight rod CD can evidently replace the bent rod CAD. Make AD longer than AC, and place unequal weights at the ends C and D (Fig. 30). Find what would be the length of the straight horizontal rod which can replace the bent rod CAD. Then measure by means of a protractor the angles α and β which the two portions CA and AD make with a horizontal rod, and prove numeri-

cally that $A\ D \times \cos \alpha = A\ E$, and that $A\ C \times \cos \beta = A\ E'$.

EXPERIMENT 59.—Obtain a piece of watch-spring from a jeweler. Bend it straight. Cut it into two equal portions about six centimetres long. Magnetize these strips equally. This can be done by stroking each strip from the center to each end by a bar-magnet, using one pole for one direction and the other for the opposite direction. Paste the two magnets on a circle of card-board so that the middle of each steel strip will lie on the center of the card-board. Suspend the card-board by means of a silk strand free from torsion. Such a strand can be obtained from a silk thread by untwisting it and moistening one of the constituent strands. If the card-board does not lie in a horizontal plane, balance it by small bits of wax. It will be noticed that the line bisecting the line joining the two poles of the same name points in the same direction as a compass-needle. Here we have substituted the magnetic pull of the earth on each of the poles for the pull of gravitation in Experiment 58.

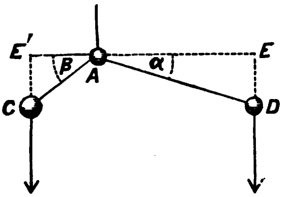

FIG. 30.

EXPERIMENT 60.—Magnetize the needles unequally by stroking one a greater number of times than the other, and observe the relation between this experiment and Experiment 58.

EXPERIMENT 61.—Screw two smooth pulleys, which can be obtained at any hardware-store, into a board placed vertically. Pass a string over these pulleys. Weight the string at each end, and also

in the middle (Fig. 31). Lay off on the line AB a line, Am, to represent the weight W_1 (if W_1 is ten grammes, the line Am can be taken equal to ten centimetres); and in the same manner a line An to represent the weight W. On completing the parallelogram, the diagonal will be found to be equal to the weight W_2. Attach two spring balances to an ordinary wooden hoop in such a manner that they can be clamped upon the hoop in any position (Fig. 32). Connect a weight, W, with both springs by means of cords, and vary the position of the spring balances $S S^1$, so that the junction O of the cords shall always pass through the center of the circle O. Find the relation between the weight W and the pull upon each balance. The hoop should be placed against a vertical wall, and a paper protractor, with its center at O, the center of the circle, will give the angles made by the inclination of the cords at O.

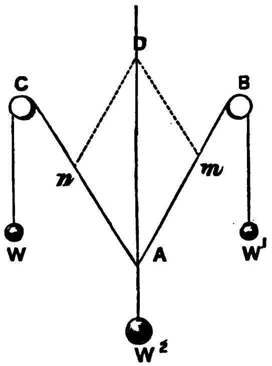

FIG. 31.

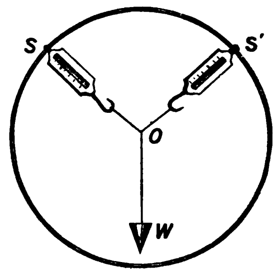

FIG. 32.

EXPERIMENT 62.—Attach a weight, w, to one end of a horizontal lever supported at its middle, A, on a knife-edge (Fig. 33). Attach another weight, w_1, to the other end by means of a string, which passes over a smooth pulley. Main-

tain the lever in a horizontal position by attaching another weight, w_2, at A'. Ascertain the relations between the weights w_1, w, and w_2 and the distances $A\ B$, $A\ C$, and $A\ A'$.

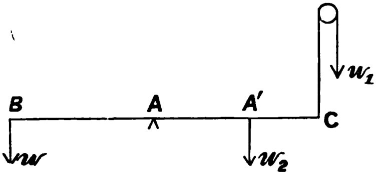

FIG. 33.

EXPERIMENT 63.—Support a magnet, about ten centimetres long, upon a cork, and place the cork upon the surface of water in the middle of a large dish. Turn the magnet so that its poles point east and west, and then release it. It will be noticed that the magnet will rotate until its poles point north and south. It will not move forward. Hence, two equal and opposite forces acting upon the magnet will produce rotation but not translation. In other words, the cork will simply revolve on its center, and this center will not move along the surface of the water. Two equal and opposite forces, acting at the end of a lever, constitute what is called a couple.

Pendulums are of various kinds. The simplest is the vertical pendulum, which is actuated by the force of gravitation.

Hang up a weight by a string and set it to swinging. Count the number of times it crosses the lowest point of its swing in a minute, and it will be found that, as long as the length of the string remains the same, the pendulum makes the same number of swings in a minute, and therefore can serve as a measure of time.

Suspend a magnet by a string attached to its middle, and set it into vibration by turning the end

that points north a little out of the north and south line, or what is called the magnetic meridian, and count the number of times it crosses the middle point of its swing, or the magnetic meridian. It will be found that this number is invariable for the same magnet, if it is not allowed to lose its magnetism. A magnet can, therefore, serve as a measure of time.*

In the case of the gravity-pendulum, we have to consider the force that draws the weight to the earth. This force is a vertical one. Hang up a number of pendulums, and they will be seen to be all parallel to each other. A spring-balance will show the same pull, in the room where you experiment, when the same weight is attached to it, at the same height, wherever it is placed. You can imagine, therefore, the room to be filled with a vast number of vertical lines of force, which pass through every inch of the floor of the room. Such a room constitutes what is called a field of force—a uniform field of force, in which all the lines of force are parallel, and the force acting along these lines is everywhere the same, and equal to a definite pull on a spring-balance. In the case of the gravity-pendulum, we have this vertical force acting upon every particle of the weight, tending to make it move along the lines of force toward the earth; it would fall vertically if it were not for the string. Hang the string to a spring-balance. When the string is vertical, the pull on the balance represents what we call the weight of the pendulum—that

* The magnetism of the earth changes slightly from year to year, and varies with the locality. It can be taken as invariable during several months. Methods of measuring it are given later.

is, the united pull of the force of gravitation on every particle of the mass of the pendulum-weight. When we draw the pendulum-bob out from the vertical by a string attached to the bob, and which is pulled in a horizontal direction, we perceive that it is necessary to exert a pull in order to hold the pendulum and prevent it falling to the vertical position. At the same time we perceive that the pull on the spring-balance is less than it was when the pendulum was in the vertical position. Since the force of gravitation is unaltered—for the same number of lines of equal force pass through the pendulum-bob as before—the force which balances the earth's pull must be the resultant of the force along the string, or its tension and the force that is exerted in a horizontal direction. These forces can be laid off, as in Experiment 61, and the resultant will be found equal to the pull of the earth on the pendulum-bob. When the horizontal pull is released, the pendulum-bob is urged along the arc of a circle by the resultant of the earth's force and the pull on the string which sustains the pendulum-bob. It is then falling down an inclined plane.

EXPERIMENT 64.—Suspend two smooth and small lead balls, one inch from each other (see Fig. 34), at the top of a smooth inclined plane. Set the circle *B* (see Experiment 49), in uniform rotation, and burn the thread at *a*. Measure the angular interval between the impressions of the ball on the surface of the revolving circle *B*. Afterward suspend the two balls at *M* and *N*. Allow them to drop in the same manner, preserving the same uniform rotation of the wheel *B*, and measure the angular interval between their impressions on *B*. It

will be found that these intervals are sensibly the same, which shows that the balls are moving with the same velocity, whether they fall down the inclined plane or down the height of the inclined plane. Allowance must, of course, be made for the retarding effect of the friction of the balls on the inclined plane. It will take, however, a longer time for the balls to fall down the inclined plane than to fall down the height of the plane. This will be made evident by suspending two equal balls, one so that it rests on an inclined plane, and the other hangs freely, so that it may fall down the height of the same plane, and burning the string between them, letting them in this manner fall upon a tin pan or other sounding material. It will be noticed that there is an interval of time after the ball which falls down the height of the plane has struck, and before the sound of the blow of the ball which has fallen down the inclined plane is heard. This also can be proved by employing an inclined plane of a sharp vertical angle, and allowing the two balls of the previous experiment to fall upon the revolving-wheel *B*. It will be found that there is an angular interval which represents the difference of time between falling down the hypotenuse, or length of the inclined plane, and the height of the plane.

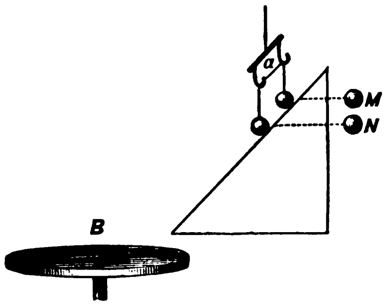

Fig. 34.

EXPERIMENT 65.—Prove experimentally that a body will fall down the vertical diameter of a vertical circle in the same time that it will fall down any chord of the circle which can be drawn from the upper end of such a diameter.

Or, Fig. 35, to prove that a ball will fall down the diameter to *D*, starting from *A*, in the same time that it will fall down *A C* or *A C'*:

Arrange the revolving-table *B* so that *A D*, the hypothenuse of an inclined plane, may be a vertical diameter of a circle, of which *A C* is a chord. Burn the thread which holds the balls, after having set the wheel *B* in revolution. It will be seen that the impressions of the blows of the two balls are along the same diameter of the circle *B*, there is no angular interval between them, and therefore no interval of time. A body will also fall down the chords *D C* and *D C'* in the same time it will descend the diameter *A D*.

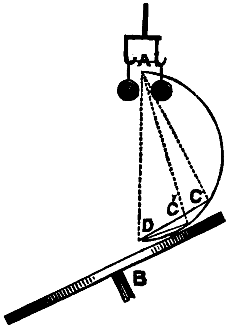

FIG. 35.

In the gravity-pendulum, the weight of the pendulum in falling down the small arc of a circle, which is always approximately equal to the length of an inclined plane, acquires the same velocity as if it had fallen down the height of this plane. When it arrives at the bottom of the plane, or at the point where the pendulum rod or string is in the same line with the direction of the earth's attraction, it has the velocity it has acquired in falling down the height it has been lifted above this point. This velocity would carry it again up another inclined

plane equal in height to the first one, if there were no resistance to be overcome, and then it would fall again, and so its motion would continue forever. The resistance of the air, however, and the friction of the thread upon its point of support brings it after a time to rest. It will be noticed that the work done, when there are no resistances in lifting the ball up the inclined plane against the pull of gravitation, is equivalent to the work done by letting the ball fall down the same inclined plane. This fact will be enlarged upon later.

EXPERIMENT 66.—To show that a pendulum moves through different lengths of arc in the same time: Suspend two equal pendulums side by side (Fig. 36). Provide the bobs of each with a small, flexible pointer, which shall just touch a surface covered with lamp-black. This surface should be that

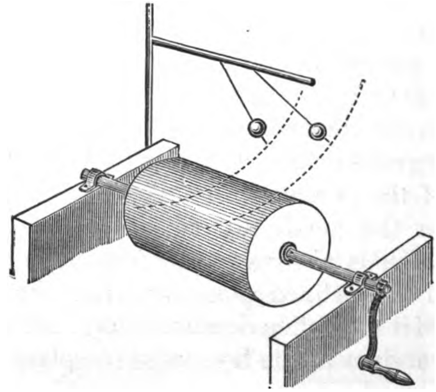

FIG. 36.

of a cylinder. Set the cylinder in rotation, and let the pendulums descend at the same instant from unequal heights. It will be found that the lines

drawn on the blackened cylinder will indicate that the pendulums passed together, and at the same instant, through the lowest point of their swing. Lines of unequal length will be drawn upon the revolving cylinder. These lines are proportional to the velocity acquired by the pendulums in falling through their arcs. Measure the length of these lines, and compare them with the velocities a body would acquire in falling vertically through the heights the pendulums have descended.

Another method of studying the law of falling bodies requires us to know the law of pendulums. This law is discussed later. We can anticipate it so far as to use the pendulum as a simple indicator of time.

EXPERIMENT 67.—Suspend a lead ball at the end of a string, set it to swinging, and count, by means of a watch, its number of swings in a certain number of ticks of the watch. A vibration of a pendulum is from the extreme point of the swing on one side of a vertical line passing through the point of suspension of the pendulum to the extreme point reached by the pendulum on the other side of this line. Having obtained the time of the pendulum, arrange the experiment (Fig. 37) so that the falling body will reach the horizontal plane passing through the lowest point of the swing of the pendulum in the same time that the pendulum-bob reaches it. This can be ascertained by quickly drawing a board, covered on its upper surface with a sheet of tin smoked with lamp-black, beneath this lowest point. In order to insure that the falling body and the pendulum-bob shall be released at the same instant, it is well to make them both

of iron, and to place them at the poles of two electro-magnets, *A B*. A thin piece of paper should

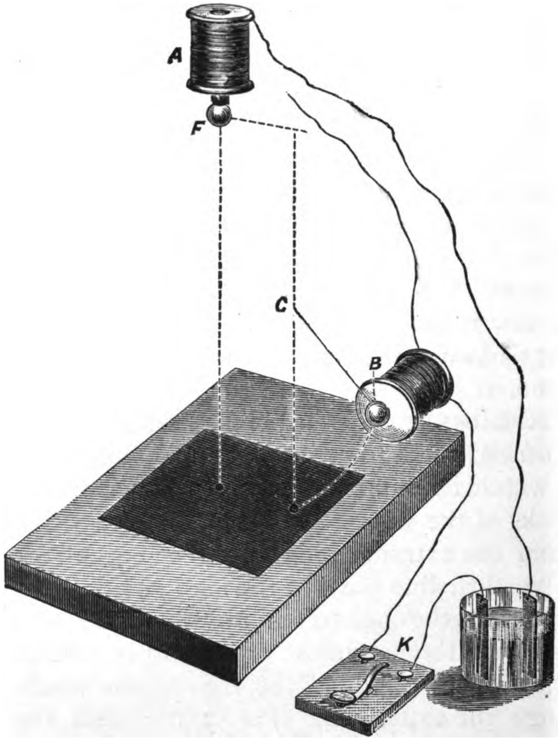

FIG. 37.

prevent their contact with the iron cores of the electro-magnets. On breaking the circuit at *K*, the balls will be released practically at the same instant.

(For description of electro-magnets and battery, see Appendix.)

With this apparatus one can find the relation between the spaces fallen through and the times of

falling. The times of swing of the pendulums of different lengths can be ascertained by counting the number of swings during a sufficiently long interval of time, and dividing the number of seconds by the number of swings.

With this apparatus prove that $s = \frac{gt^2}{2}$, in which $s =$ space in feet, $t =$ time, and $g = 32.2$ feet.

Find, also, the relation between the distance the body F falls and the length, CB, of the pendulum.

In order to obtain the final velocity of the falling body, suspend two small lead balls with a small piece of iron or steel driven into them—one vertically above the other, by means of two electro-magnets, and distant from each other by a small measured interval. Arrange the revolving cylinder of Experiment 66 beneath them, so that, when the electrical circuit which contains both the electro-magnets is broken, the balls will strike the revolving cylinder one after the other. Arrange a tuning-fork of known number of vibrations, which shall record its rate upon the revolving cylinder. The number of the vibrations of this fork between the marks made by the two falling bodies will give the rate of the uppermost ball while traversing the known distance between the two falling bodies.

The prong of the tuning-fork should be provided with a little flexible wire, and the fork should be mounted so that, when the fork is excited by a bow, this little wire shall draw a sinuous line on the revolving cylinder. If we know the number of vibrations that the fork makes per second, by counting the sinuosities or waves between the marks made

by the falling bodies, we shall obtain the time of arrival of one ball and that of the other, and in this way obtain the rate of acceleration during the last second of the descent. In this way prove that $v^2 = 2gs$, in which v is velocity, g is 32.2 feet, and s is space fallen through in feet.

We have thus far verified the following expressions:

In uniform motion, space = vel. × time, or $s = vt$; velocity = acceleration × time, or $v = gt$.

In accelerated motion
$$s = \frac{gt^2}{2}$$
$$v = \sqrt{2gs}$$

Force = mass × acceleration, or $F = \frac{W}{g} \times f$, in which $W =$ weight, and $g =$ acceleration of gravitation = 32.2 feet per second = 981 centimetres per second. Thus far we have measured only the acceleration of gravitation. Let us now examine the question of acceleration in general.

In order to do this, we generally are obliged to modify the motion of bodies by certain mechanical arrangements which involve friction and the idea of inertia.

The ratio of the friction to the pressure, perpendicular to a surface, is called the co-efficient of friction. For example, if our weight is 100 pounds and the friction amounts to 10 pounds, the co-efficient of friction is $C = \frac{10}{100} = \frac{1}{10}$. To draw a sled weighing 100 pounds along a surface of snow, the co-efficient of friction being $C = 0.04$, we require a force of $F = 0.04 \times 100 = 4$ pounds.

EXPERIMENT 68.—Arrange the following experiment:

A weight, W is drawn along a horizontal oiled or greased surface, by means of the tension T, on a thread which passes over a small pulley. Ascertain, in the first place, the coefficient of friction between the weight W and the greased surface. This can be done by attaching a spring-balance to the weight W and ascertaining what pull is needed to just move it. This pull, or force, is equal to the friction, and the ratio between this force and the weight W is called the co-efficient of friction.

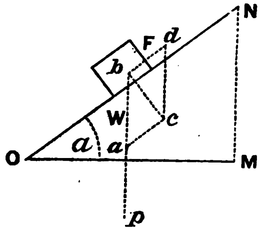

FIG. 38.

EXPERIMENT 69.—Attach two boards together at one end by a hinge (Fig. 38). Place one board on a table and elevate the other until a weight, W, shall just begin to slide down the greased inclined plane. Measure the angle a of this inclined plane by means of a protractor, or by finding the ratio of the height MN to the base OM; this will be the natural tangent of a, and can, therefore, be obtained from a table of natural tangents.

The pressure on the perpendicular plane will be $ab \cos a$ (since the angle abc is equal to a, for ab is perpendicular to OM and bc is perpendicular to ON. Two angles are equal whose sides are mutually per pendicular). The friction is proportional to this pressure, and opposes the component of W, or ac, which would urge the body down the inclined plane.

Hence we have $C\, W \cos a = F$, or $C = \dfrac{F}{W \cos a}$.

But the ratio $\dfrac{F}{W} = \dfrac{ac}{ab} = \dfrac{MN}{ON} = \sin a$. Hence $C = \dfrac{\sin a}{\cos a} = \tan a$.

This value of C should agree with that obtained by measuring the force necessary to just move the weight along a horizontal surface which is covered with the same substance as the inclined plane. It is well to test the truth of the trigonometrical relation, cosine a and $\sin a$, by actual measurements.

EXPERIMENT 70.—Let P be the force necessary to draw the weight W along the greased surface. The motive force will then be $F = P - C\,W$, in which C is the co-efficient of friction.

The mass moved will be $\dfrac{P+W}{g}$, in which g is the acceleration of gravitation. The force, however, is always the mass moved multiplied by the acceleration, or $F = Mf$. The mass, however,

$$= \frac{\textit{weight}}{\textit{acceleration of gravitation}} = \frac{P+W}{g}.$$

Hence $F = \left(\dfrac{P+W}{g}\right) f$, or $P - C\,W = \left(\dfrac{P+W}{g}\right) f$, or $f = \dfrac{P - C\,W}{P + W}\, g$. We also have $s = \dfrac{f t^2}{2}$, or $\dfrac{2s}{t^2} = f$.

Hence, $\dfrac{2s}{t^2} = \dfrac{P - C\,W}{P + W}\, g$.

Ascertain from direct experiment what value of g will satisfy this equality. To do this, suspend a simple pendulum, whose time of vibration you have ascertained, so that it shall be in the same vertical

plane with the string to which the weight P is attached. Attach a small bit of wire to the bob of the pendulum, arrange the position of P so that it will fall through a certain height in the same time that the pendulum executes one half its swing.* In this

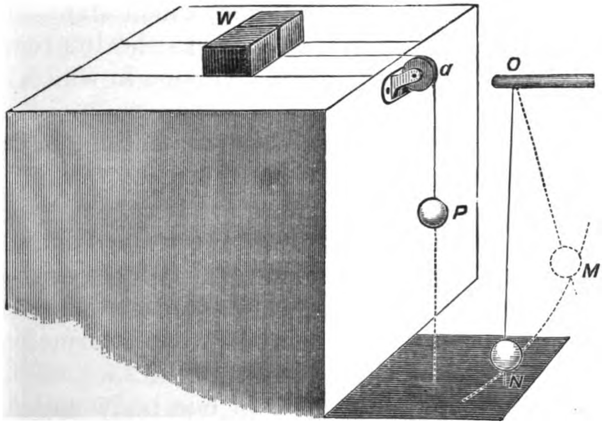

FIG. 39.

case the weight P and the pendulum-bob will touch a plate of smoked glass or paper, which is drawn horizontally by the hand beneath the lowest point of the swing of the pendulum, at the same time; the time of the pendulum will then give us the value of t in the formula, which is one half the time of the pendulum, and the space fallen through by the weight P in this time is what had been arranged. In this experiment we neglect the friction and inertia of the pulley a, and also the rigidity of the

* This experiment requires two persons—one to release the pendulum and the weight P at the same time, and another to draw the smoked surface.

string. The friction of the pulley should be reduced by oiling, and a strand should be used to connect *W* and *P*. Two electro-magnets, however, can be used, as in Experiment 67.

Our experiments on the motion of bodies falling under the influence of the force of gravitation illustrate the second law of motion, which states that change of motion is proportional to the impressed force, and takes place in the direction in which the force is impressed.

By motion is understood momentum, or the quantity of matter moved multiplied by the rate at which it moves.

In order to clear our ideas, certain definitions are necessary. We can not define force without having glanced over the whole subject of motion. It is necessary for us to perceive what is termed the relativity of all phenomena. In physics we are constantly comparing the state of one body with the state of another, both in regard to volume and in regard to motion. We get our ideas of force by the work which is necessary to do in order to resist a change of position of the material particles of a system. We have defined the "unit of force to be that force which, acting on the unit of mass during the unit of time, generates the unit of velocity." We also know that the total effect of a force in communicating velocity to a body is proportional to the force, and to the time during which the force acts. We have spoken of an impressed force as an impulse. By impulse is meant the product of the force by the time during which it acts — understanding by the word force its mean intensity.

By the definition of a unit force, we perceive that

the force is expressed as a velocity. Since our exact knowledge of nature depends upon measurements, it is very important that we should have a common standard for the measurement of length, and also of mass, and of force. The system which we shall use is called the centimetre-gramme-second system, or the *C. G. S.* system. The advantage of the metric system in determining distance and volume we have already dwelt upon. Since this system often involves the use of large numbers, it is usual to express the results by certain numbers multiplied by ten raised to a power; thus, $45,000,000 = 45 \times 10^6$.

The *C. G. S.* unit of force is called a *dyne*. A dyne is the force which, acting upon one gramme during one second, generates a velocity of one centimetre per second.

The *C. G. S.* unit of work is called an *erg*. It represents the work done by one dyne through a distance of one centimetre.

The unit of energy is also the erg.

Work is expressed in gravitation measure as follows: 1 gramme-centimetre is equal to g ergs (g being the acceleration of gravitation, or the standard by which we measure force, $g = 980$ centimetres for latitude of Paris). One kilogrammetre is equal to 100,000 g ergs. One foot-pound is 13.825 g ergs, or taking $g = 980$, is 1.355×10^7 ergs. The unit rate of working is 1 erg per second; a watt is 10^7 ergs per second.

The word *force* is often used in a wrong sense for energy, and also for the power of doing work. An engine is estimated by the rate at which it will lift a given weight. We can speak of the power of

a steam-engine or the power of a horse, but we can not use the expression "force of a steam-engine," for in our idea of force we do not also include the exertion of the force through a space. When we do include this latter idea we are really speaking of work.

We define work as follows:

"Work is the act of producing a change of configuration in a system in opposition to force which resists that change."

Energy is the capacity of doing work.

"When the nature of a material system is such that, if, after the system has undergone any series of changes, it is brought back to its original state, the whole work done by external agents on the system is equal to the whole work done by the system in overcoming external forces, the system is called a *conservative system.*"

General Statement of the Principle of the Conservation of Energy.—"The total energy of any material system is a quantity which can neither be increased nor diminished by any action between the parts of the system, though it may be transformed into any of the forms of which energy is susceptible" (Maxwell's "Matter and Motion," p. 103).

The word *stress* is much used in modern physics to denote the mutual action between two portions of matter. The tension on a rope is a stress, or the pressure between two bodies. Stress implies action and reaction, for there must always be an opposite effect opposing tension. Suppose that two persons pull against each other by means of a rope, there is a stress upon the rope. Take away one of the persons, and the reaction disappears. The pull on the rope is the force exercised

by only one of the persons, for we could replace one of the persons by the resistance opposed by a post to which the rope may be attached. The stress or tension is the same throughout the rope.

In the case of gravitation, electricity, and magnetism, the bodies act upon each other without any visible means of connection. The stress appears to be exercised by means of some medium in which the bodies are placed. Even in the case of the attraction or repulsion of magnets we find that action and reaction are equal to each other.

If instead of a unit mass we suppose that equal forces are applied to every unit of mass of the system, the total force acting upon the whole mass would be the mass multiplied by the velocity which any one of the equal forces would give one of the units of mass. We express this by the equation $F = M V$. This is often called the equation of momentum. M represents the number of units of mass, and V the number of units of velocity. The direction of the momentum is that of the velocity, and, since we estimate velocity by a reference to two epochs, we must also estimate momentum by a reference to some point. Force is sometimes said to be momentum. Momentum can only be estimated by its changes. We conceive of something which produces these rates of change, and this something is what we call force.

Thus we say that "the change of momentum of a body is numerically equal to the impulse which produces it, and is in the same direction" (Maxwell's "Matter and Motion," p. 75).

CHAPTER VII.

MOMENTS OF INERTIA.

In the case of the magnetic pendulum we have the same species of motion as in the case of the vertical or gravity pendulum. The magnetic pendulum, however, is not a vertical one. It moves in a horizontal plane. The force acting upon it is the pull that the north pole exerts upon the south pole of the suspended magnet. An equal pull is exerted on the north pole of the magnet by the south pole of the earth. These pulls upon magnets suspended in the same room beyond each other's influences are all in the same direction; for all ships' compasses, free from the influences of iron, and at the same place, point to the north pole. This is so far from our habitable regions that the lines of magnetic force converging to it are approximately parallel. To prove that the magnetic force is equal everywhere along these lines of magnetic force, as in the same room, where there are no masses of iron, make a magnet, suspend it and set it in vibration. It will be found that it will make the same number of vibrations wherever it is placed in the same room (where there are no other masses of iron or steel). We can therefore picture to ourselves lines of magnetic force which are everywhere parallel,

and along which the magnetic pull is the same. Great care is taken in magnetic measurements to have the magnetic pendulum vibrate in such a field of uniform magnetic force. There is little difficulty in obtaining a uniform field of gravitative force in which to swing our gravity-pendulum, but it is generally very difficult to obtain a field of uniform magnetic force. If the field of magnetic force is not uniform, the time of the magnetic pendulum would vary, for at one time it would be falling down an inclined plane under the action of one pull, and at another time down another inclined plane under another pull, and all uniformity would be lost.

If the earth's pull on a pendulum should increase, our gravitation-pendulums would move faster, for the force urging them down their respective heights would be greater.

If the earth's magnetic force should increase, our magnetic pendulum would swing faster. We have no method of increasing the earth's gravitative force, and therefore there is no way of increasing the time of swing of a gravity-pendulum of which the length is maintained constant.

We can not increase the earth's magnetic force, but we can make a field of magnetic force of any strength we choose, in which we can place our magnetic pendulum and increase or decrease its time of swing at pleasure. This artificial field, however, must be a uniform one, such as we have previously described.

EXPERIMENT 71.—To make a uniform field of magnetic force in which a magnetic pendulum swings faster than in the earth's magnetic field.

Having sprinkled some fine iron-filings upon a piece of card-board, place two magnets in such a manner that the iron-filings will arrange themselves in straight lines between the poles, as in the figure. Suspend a short magnet, *A* (Fig. 40), made of a bit of watch-spring, between the poles of these magnets: it will be in an approximately uniform field of

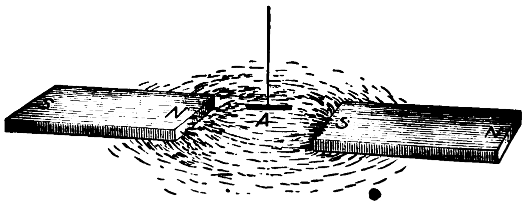

FIG. 40.

magnetic force. On setting it into vibration by suddenly bringing another large magnet near it, it will be seen that it swings much more rapidly than in the earth's field. Instead of the permanent magnets, two electro-magnets can be used.

Second Method of ascertaining the Law of Falling Bodies.—If we could make a gravity-pendulum or a magnetic pendulum vibrate very fast, and could also count these vibrations, we could use such a pendulum to estimate small intervals of time. The shortest gravity-pendulum we can make, however, will not enable us to estimate tenths of a second; and we can not follow the movements of a quicker moving magnetic pendulum by the eye. A tuning-fork which sounds a comparatively low note, as the middle C on the piano, affords us a convenient means of measuring small fractions of a second.

If, now, we could arrest a swiftly-moving mag-

netic needle, which has been put in a strong magnetic field, at any point of its swing, and if we knew what interval of time this portion of the swing corresponded to, we could measure the time of falling of a body by setting a magnetic needle free, which is held at a certain deflection at the instant the body begins to fall, and stopping the needle in its swing the instant the body has fallen through a certain height. This can be accomplished as follows:

The tuning-fork *F* (Fig. 41) is firmly fixed in a vise, *V*, and adjusted until the style, or light pointer attached to one of its prongs, just touches a plate of smoked glass, which can slide beneath it. This plate of glass is placed upon a piece of board, which slides between guides, *G G*. This piece of board also carries a piece of stiff brass, *P*, which slides with it, and touches *m* or *m'* merely with its point. The extent of the slide of *P*, and the

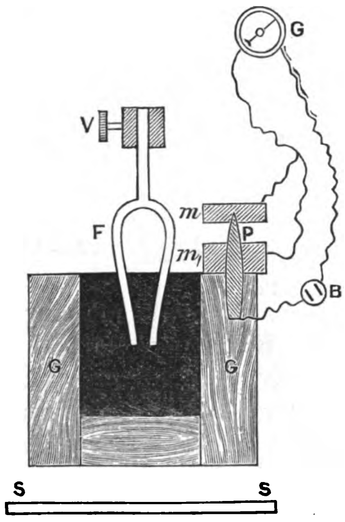

FIG. 41.

board to which it is attached, is limited by the stop *S S*. The point of the piece of brass rests upon a piece of brass, *M*, when the slide is drawn back. The brass pointer leaves the brass *M*, crosses an interval of wood or paper, or other non-metallic substance, and rests upon another brass plate, M^1. If,

now, the tuning-fork should be set into vibration by a bow, and the slide should be drawn back to the stop *s s*, the tuning-fork will draw a sinuous curve upon the smoked glass, whose length will evidently be equal to the distance the point of the brass piece *P* has traveled. By laying out on this sinuous line the distance of the starting-point of the point of *P* from the edges of the brass pieces *M*, M^1, we can evidently obtain, by counting the number of vibrations registered on the smoked glass between these brass pieces, the time it has taken to cross the interval from one brass piece, *M*, to the other, M^1. In the accompanying Fig. 42, we see that there are two complete vibrations, and a fraction of another one, which must be carefully estimated.

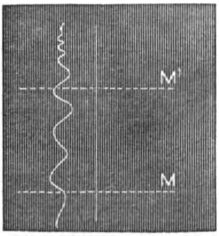

FIG. 42.

Let us now connect the two brass pieces *M*, M^1 with one pole of a battery, *B*, and the brass pointer with the other pole of the same battery, passing the current through a galvanoscope, *G*. It is evident that, when the brass pointer touches the brass *M*, the current is closed, and a current passes through the circuit. This circuit is broken when the pointer leaves the brass *M*, and is made again when it has crossed the interval between *M* and M^1 and touches M^1. By regulating the interval between *M* and M^1, one can make the interval what one chooses, and the tuning-fork will give the time of crossing the interval.

Let us now look at the action of the galvanoscope *G*. This is made as follows: A piece of board, about five inches wide, one inch and a quarter thick,

and eight inches long, has two grooves cut in it. Four rounded cleats, *c c*, are glued above these grooves, as in Fig. 43. Over these cleats are wound several turns of wire continuously, as if one were winding thread on a spool, leaving a space, however,

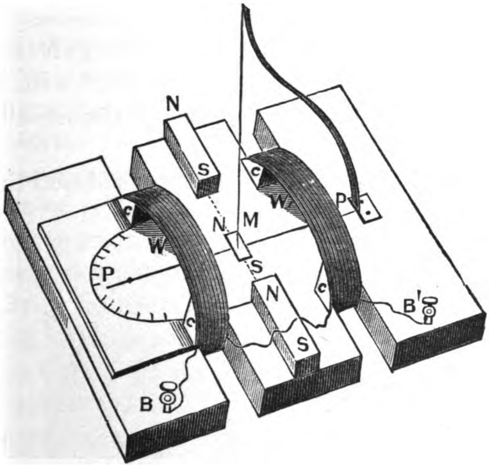

FIG. 43.

between the two equal windings, *W*, W^1 (Fig. 47). The ends of the windings are brought to binding-screws at *B* and B^1. Suspend a small magnet, *M*, made from a bit of watch-spring, and provided with a light, long pointer, *P*, P^1 (which is at right angles to the magnet), by a silk fiber in the middle of the space between the two windings, as is shown in the accompanying figures.

Place two equal magnets north and south of the little suspended magnet, in order to increase the magnetic field in which it swings. Place a plate of looking-glass under the long pointer, at one

side, and cut out the graduation of a paper protractor, or any graduated circle, and paste it upon the looking-glass, so that the center of the circle from which it was drawn shall be directly at the center of the two windings, and therefore under the center of the suspended magnet. The looking-glass is to avoid parallax in reading the excursions of the pointer attached to the magnet. This galvanoscope should be placed under a box provided with a glass lid, *L* (Fig. 44), in order to protect the needle from currents of air.

When the circuit of the battery is closed through

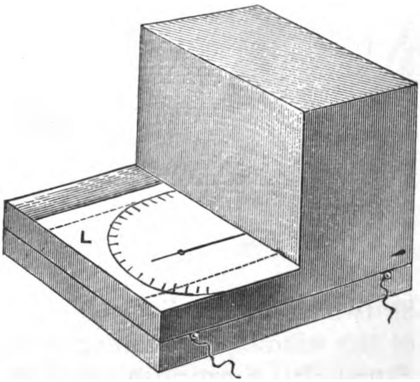

FIG. 44.

this galvanoscope, the needle is deflected, and the pointer attached to the magnet indicates a certain angle on the graduated circle. This is the case when the pointer rests upon the brass piece *M*. When the pointer leaves the brass *M*, at that instant the circuit is broken, and the needle rapidly swings toward its position of rest. When, however, the brass pointer touches the brass M^1, its fall is ar-

rested, it receives a sudden check, and swings back to its first deflection. The time of its fall, from this first deflection to the point where it receives a check, is registered by the tuning-fork in the manner we have indicated above. With a suitably constant cell, like a Daniell cell (see Appendix), the times of small breaks in an electrical circuit can be read off from the angles through which the needle of the galvanoscope falls, having first ascertained the value of these angles in time by means of the tuning-fork.

Let us now apply this apparatus to determining the laws of falling bodies, not only in the hope of reaching more accurate results than by our former apparatus, but also to obtain a knowledge of the use of electrical apparatus.

EXPERIMENT 72.—To find the time it takes a body to fall under the influence of gravitation:

Suspend a ball, *C*, from a bit of spring brass at *A* (shown at *a*, in Fig. 45), which completes the electrical circuit through *G* and the battery *B*. When the spring which holds the falling body *C* is burned, the latter in falling breaks the circuit at *A* (Fig. 46), and, in striking at *D*, closes it again.

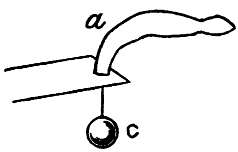

FIG. 45.

The break at *D* should be arranged so that it will be kept closed after the falling body strikes it, otherwise the needle of the galvanometer will not receive a decided check in its fall to a position of rest. The height through which the body falls can then be compared with the times of falling by knowing the values of the angular deflections of

the magnet in times. This can be determined previously, in the manner we have shown, by the tuning-fork, provided that the electrical current is maintained uniform. This will not be difficult, if a Daniell cell is employed.

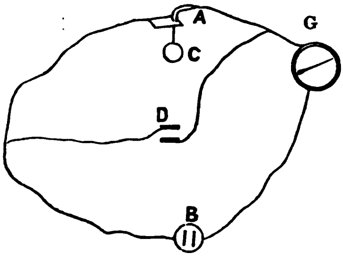

FIG. 46.

EXPERIMENT 73.—Prove that the squares of the velocities attained by a body falling through different heights are proportional to the heights.

This can be done by arranging the electrical connections so that the falling body shall break the circuit after it has been falling a certain time, and shall make it again at any suitable time. Thus, when the falling body strikes I (Fig. 47), it breaks the electrical circuit, and, after falling through the distance I, I^1, it makes the circuit again at I^1, and keeps it permanently closed. The breaking at I should not sensibly impede the fall of the heavy falling body. This can be accomplished by making a delicate connection at I.

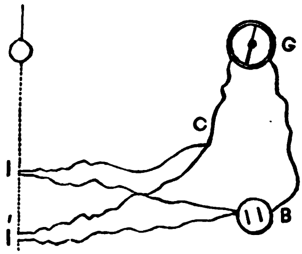

FIG. 47.

In one case the electrical circuit is $I\ C\ G\ B$. When the falling body rests on I^1, the circuit is closed through $I^1\ C\ G\ B$. The distances $I\ I^1$ should be small, for the velocity is not uniform; but we

suppose that the velocity during the last two or three inches of a fall is uniform. This is approximately true. In this way we can prove that $\frac{v^2}{v_1^2}=\frac{h}{h_1}$, in which v and v_1 are velocities, and h and h_1 spaces or heights.

There is still another simple apparatus by which the same law can be tested; the error arising from friction, however, has to be carefully estimated. It is as follows:

EXPERIMENT 74.—Adjust a tuning-fork of known rate of vibration in a vise (Fig. 48). Allow the tuning-fork and the vise to slide in vertical guides, which are carefully greased so as to avoid friction as much as possible. Arrange a smoked glass at suitable distances, so that the stylus or point upon the tuning-fork shall just touch the smoked glass in the last few instants of its fall, and register its vibrations for a measured space. If the slide containing the vise and tuning-fork is sufficiently weighted, and the friction in the guides is made small, the final velocity of a falling body, which, in this case, is the apparatus for measuring the time, can be readily measured. The final record will be

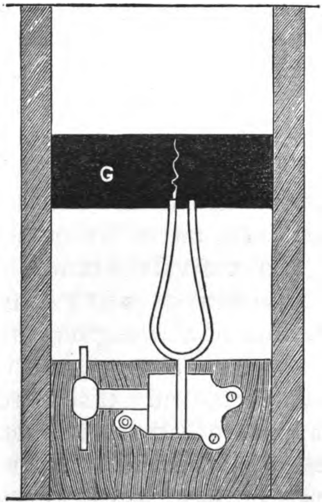

FIG. 48.

of the nature indicated in the accompanying figure. By laying off a measure-line upon this sinuous curve, the number of vibrations of the tuning-fork, made while it passed through this distance, can be ascertained, and from this the velocity. Instead of a tuning-fork an elastic rod can be employed, and its rate of vibration be determined afterward by a tuning-fork of known rate.*

EXPERIMENT 75.—*Laws of Inclined Planes.*—By inclining the guides, the laws of the fall of bodies down inclined planes can also be tested.

We have given various methods of determining the laws of falling bodies, both to obtain practice in physical experimentation and to obtain clear ideas of energy and of work. We have seen how important it is to have a uniform standard of time, in order to know how long an interval elapses between one occurrence and another. But why do we wish to know the time between two events? Is it not because some effort must be made—some work must be done, either by ourselves or by others, during these intervals of time? A steamer sometimes runs for several days in a dense fog, the number of revolutions of the engine is recorded, and the time during which these revolutions have been maintained is ascertained. If sixty revolutions of the screw per minute will drive the steamer ten miles in an hour, the distance the steamer has gone in several days can be approximately ascertained. This is called dead-reckoning. The velocity of the screw and the rate of time of the clock or watch are both supposed to be uniform. If either varied, the steamer might be driven upon dangerous rocks in a fog.

* See Müller's "Physik," p. 254.

Hence the necessity for a uniform standard of time. Behind this estimate of time and velocity, however, is an (unconscious, perhaps) estimate of the work that the engines can do during the same interval of time. It is this work that conveys the steamer. When we carry a weight up a steep hill, the time we take, and therefore the velocity, becomes of great importance, when we consider the weight which we carry. We should prefer to engage to roll small cannon-balls up to a certain point on an inclined plane, rather than cannon-balls three times as great, at the same velocity. In order to shoot an arrow vertically, so that it may attain a certain height, we must exert a certain pull upon the bow-string. In order to increase this height, we must increase our pull on the string. Now, the arrow, having attained a certain height, will return in a certain time and with a certain velocity. If we neglect the resistance of the air, the arrow or the rifle-ball will return with the same velocity with which it was shot forth. We have seen that the square of this final velocity is proportional to the height through which the body falls, or $v^2 = 2gh$, or $h = \frac{v^2}{2g}$.

In order therefore to shoot an arrow to a certain height, we must do a certain amount of work upon the bow-string, or burn a certain amount of powder in a rifle in order to communicate the velocity which will carry the missile to a certain height in a certain time. Behind the exhibition of velocity, therefore, is an exertion of work. We could measure the muscular work done by different people in pulling a bow by the height to which the missile rises, or,

by the velocity the missile acquires in falling from this height. In carrying a heavy weight up a flight of stairs, we are conscious of working against the force of gravitation at every instant. If we carry a ten-pound weight up a flight of stairs twenty feet high, we feel that the work we do is proportional to the number of feet we rise, for the force of gravitation is a vertical one, and we are always advancing along lines of vertical force against the pull along these lines. In rising above the earth, the work we do is proportional to the mass we lift, and to the pull along these lines of force, or what we term the force of gravitation, and to the distance we pull against this force, or the height we lift the weight.

The work is proportional to nothing else. It does not matter whether we are all day or only one instant in lifting the weight.

Hence we say that the work done is equal to the mass lifted, multiplied by the pull of the force of gravitation upon every particle of it, in a uniform field of gravitative force, and finally multiplied by the distance we pull against gravitation, or the height through which the mass is lifted. To express this in letters, let $w =$ work, let $m =$ mass, let $g =$ gravitation, pull, or force, let $h =$ height or distance passed through against the vertical pull of gravitation: then $W = m\,g\,h$.

Now, we have said that it does not make any difference whether we take a day to lift the weight or one minute, as far as regards the work we do in lifting this weight against the force of gravitation. If we let this weight fall from the height of twenty feet it will acquire the same velocity when it reaches

the earth, whether we take an hour to lift it or only one minute. It would take a certain muscular pull if this weight were in the shape of an arrow, to pull a bow-string in order to give the weight the velocity which would enable it to reach the height h. The work we put into this muscular pull to overcome the pull of gravitation, through this space of h, is evidently the equivalent of the work that must be done against this pull in mounting the stairs, and conveying the weight to the height of h. In estimating the work of carrying this weight once to the height h against gravitation, we find that by running up with it or proceeding slowly, we can not do more than is expressed by the muscular pull on the bow-string which will urge the same weight to the same height. If, however, it is necessary to carry the same weight up twenty feet sixty times a minute, or to shoot an equivalent weight from a bow sixty times a minute, the total amount of work will be sixty times the work that is done in conveying the same weight once up the same height.

Let us see now what work will be done by shooting a ball up a certain vertical height, h, with a certain velocity, v.

Since the body in falling through this same height will require the same velocity, v, with which it started, we have $v^2 = 2gh$, or $h = \frac{v^2}{2g}$.

But the work $w = mgh$. If, therefore, we multiply h by mg, we have work,

$$w = mgh = mg\frac{v^2}{2g} = \frac{mv^2}{2}.$$

Hence we see that the work is proportional to the mass moved and to the square of the velocity with

which it is shot forth. On the other hand, we see that the body, in falling through the height h from a position of rest, will acquire a velocity of v. If it could be arranged that this falling weight—supposed to be inelastic—should meet a similar inelastic weight just being shot forth from a bow at the foot of the height, neither would move—since both are moving with the same velocity in opposite directions, and the effort of one weight in rising would be exactly counterbalanced by the energy which the other has acquired in falling. Hence the work done by the falling weight would be equal to the muscular pull given to the bow, acting through a certain distance, to urge the same weight up the height h.

We see, therefore, that a body falling down an inclined plane would acquire the same velocity as if it fell from the height of the plane, for the work of carrying a body up the length of the inclined plane is the same as lifting it up the height of the plane.

The action of a pendulum consists in alternately lifting a weight up an inclined plane and letting it fall down this plane. The work that it does consists in lifting the pendulum up the height of the inclined planes whose lengths are the small chords of the arc in which it moves, against the pull of gravitation. If h represents the height to which the pendulum-bob is lifted at the end of one excursion, the work done, or $w = mgh = \frac{mv^2}{2}$, this would be the work developed by the pull of the pendulum-bob down the height of the inclined plane, or down to the foot of the length of the inclined plane.

Since the gravity-pendulum is employed to ob-

tain our standard of uniform time, and can be used to measure the work done by falling bodies, and the magnetic pendulum, as we shall see later, is used to measure electrical energy, we shall examine the motion of pendulums more closely.

EXPERIMENT 76.—Hang up a ball of suitable weight by a string—a leaden ball, for instance, by a fine black silk thread—and count the time of its vibrations.

In order to obtain the time of a pendulum, count the beats of a clock (or notice the number of seconds by a watch) between the time when the pendulum passes through the lowest point of its arc at the same instant that the clock beats and the next time when the swing and the beat are again together. Thus, suppose that this interval is 50 seconds. It has taken the slower pendulum this length of time to lose one vibration, hence its time will be $\frac{50}{49} = 1.020$ seconds. Take the mean of a number of such observations. This method of coincidences, it will be noticed, is a very accurate one, for, if the mean had been 52 instead of 50, we should have had $\frac{52}{51} = 1.019$, which differs from the preceding only by $\frac{1}{10}$ of one per cent.

Then alter the length of the pendulum, and again obtain its time of swing. Prove in this way the formula $t = \pi\sqrt{\frac{l}{g}}$ in which t is time of swing.

$\pi = 3.1416$, $l =$ length in centimetres, $g =$ force of gravitation (980.8 in the latitude of Cambridge, Massachusetts, expressed in *C. G. S.* units).

The formula $t = \pi\sqrt{\frac{l}{g}}$ can also be proved in another way. The method demands a knowledge of the action of forces directed toward a fixed center. The motion of a weight whirled around at the end of a string is the commonest illustration of a body moving in a curve under the action of a force constantly directed toward a fixed central point.

When we swing a stone, attached to a string, around our head, we are conscious of a pull on the string, the stone by Newton's first and third law of motion tends to preserve the direction of its motion at any instant, and we are compelled, in order that the stone shall move in a circle, to exert a tension on the string. The reaction to this pull or tension is what is often called centrifugal force. The tension on the string is the centripetal force. It will be seen that the centrifugal force is merely the tendency of the stone to continue in its line of motion, away from the hand, and is the reaction to the centripetal force.

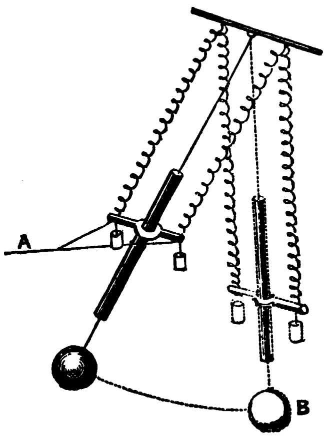

FIG. 49.

EXPERIMENT 77.— Suspend a weight at the end of a wire, about two metres long (Fig. 49). Slip a small glass rod over a certain portion of the wire, and confine it to the wire, so that it may move as a part of it. Slip upon this glass rod a small

ring, which is supported by two small spiral springs. Attach to this ring suitable weights, and fix the spiral springs to a suitable support. Blacken the glass rod by carefully moving it over a lamp which is smoking. Draw the pendulum from its point of rest to a measured height, hold the springs by means of two strings connected at A, and then burn the thread at A with a match. The small weights will acquire the velocity of the pendulum at a certain point, which is the lowest point marked on the blackened glass by the ring. This is when the pendulum is passing through the lowest point of its swing. The centripetal force elongates the spiral, and the amount of this force can be measured by ascertaining what weight will extend the spring to the mark on the blackened glass. The weight of the pendulum-bob B should be much greater than the weights attached to the spirals.

Prove that this force is proportional to the mass, the velocity being the same.

Also to the square of the velocity, the mass being the same.

Also varies inversely as the radius of the arc through which the weight swings—

$$\text{or } F = \frac{mv^2}{r}.$$

The velocity of any point of the pendulum B, at the lowest point of its swing, can be obtained by previous methods. The length of the pendulum B can be changed by lengthening the wire to which the bob is attached.

We can now prove the formula $t = \pi \sqrt{\frac{l}{g}}$ as follows:

EXPERIMENT 78.—Suspend a heavy ball from a wire about one metre long, and set it to swinging around a circle drawn with chalk upon the floor beneath it. It thus constitutes a conical pendulum which will be found to execute all its swings approximately in the same time. There is, therefore, some simple pendulum which will execute its two swings in the same time that this conical pendulum moves once around its circle. The conical pendulum can be made to move around this circle uniformly. The space described is $2\pi r$, or the circumference of the circle around which it moves. In uniform motion we always have—

$$\text{Space} = \text{velocity} \times \text{time},$$

$$\text{or velocity}, = \frac{\text{space}}{\text{time}} = \frac{2\pi r}{2T}$$

(T being the time of a simple pendulum, which makes two swings while the conical pendulum makes one).

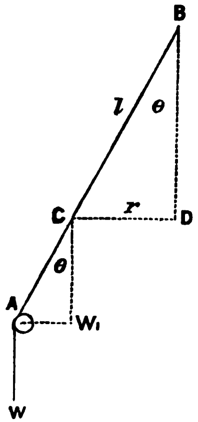

FIG. 50.

For small circles of swing the tension along the wire of the conical pendulum does not differ sensibly from its tension when the wire AB is vertical. This can be proved by attaching a spring-balance to C (Fig. 50), and connecting this balance with a cord, which passes over a smooth pulley, and has a weight, W, attached to its other end. This tension is equal to the pull of gravitation, or the mass multiplied by the force of gravitation, or mg.

We have proved that the centripetal force tending to draw a body revolving in a circle of radius r toward the center of the circle is $\frac{m v^2}{r}$. This is balanced by the reaction, or, as it is often called, the centrifugal force, which can be represented by the line $A W_1$, if $A C$ represents the tension or weight $m g$. We can prove directly that

$A C \times \sin\theta = mg \times \frac{r}{l}$ (since angle $A C W_1 = C B D$)

$$= A W_1 = \frac{m v^2}{r},$$

$$\text{but } v = \frac{2 \pi r}{2 T} = \frac{\pi r}{T},$$

$$\text{and } v^2 = \frac{\pi^2 r^2}{T^2};$$

$$\text{hence } m g \frac{r}{l} = \frac{m \pi^2 r^2}{r T^2}, \text{ or } T = \pi \sqrt{\frac{l}{g}}.$$

In our experiments thus far, we have supposed the whole weight of the pendulum to be in the bob, and we have neglected the weight of the suspending wire or rod. In many cases, however—like the swinging magnet, for instance—the weight of the pendulum is distributed throughout its entire length. The movement of such a pendulum is very different from a pendulum in which all the weight is collected in the bob, as in the pendulums with which we have experimented.

EXPERIMENT 79.—Suspend an iron or brass rod (Fig. 51), two feet in length, from a very short string attached to one end, and having made a lead weight, or having filled a tin vessel with sand, so that the weight of the lead, or of the filled vessel, shall equal

the weight of the metallic rod, suspend this equivalent weight by a light string or wire, and lengthen or shorten this suspension until the two pendulums swing together in the same time. It will be found that the equivalent pendulum, or simple pendulum, in which all the weight is collected in the bob, has a shorter length than the metallic pendulum.

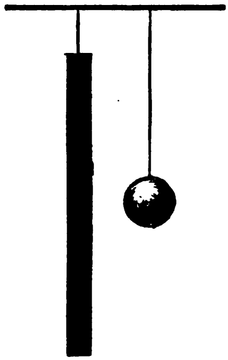
FIG. 51.

This is indirectly an illustration of the conservation of energy; for the work the metallic-rod pendulum does in each swing is the weight of each little particle multiplied by its fall toward the earth. Each small particle falls a different distance. The work done by all the particles must lie between the work done by the fall of the particles near the lower end of the rod and those at the upper end. There is, therefore, some point where, if the weight of the entire rod were collected, its work in descending an arc, or the height of an inclined plane, would be equal to the work of all the particles in descending various different arcs. The length of a simple pendulum, whose time is equivalent to that of a compound pendulum—like a uniform metallic cylinder, or any irregularly shaped body—gives the radius of the arc, down which the total collected weight of the compound pendulum can descend, and perform the same work that all the little particles of the compound pendulum perform in descending their various arcs.

The length of the equivalent simple pendulum depends upon the radius of gyration. Instead of finding this radius of gyration by experiment, as we have done, it can be calculated. This calculation requires the use of a term called the moment of inertia, which is a very important term, and requires careful explanation. Remembering, from the illustrations just given, that we obtain the length of a simple pendulum, whose time of vibration is equal to that of a compound pendulum, for the purpose of estimating the work done by all the particles of a compound pendulum in rising and falling against the force of gravity, we shall see that the term "moment of inertia" is necessary to estimate the work done by all moving bodies.

We know, from the law of the lever, that $Fl = F'l_1$, where F and F' are parallel forces, and l and l_1 are the distances of their points of application from the fulcrum A (Fig. 52). The products Fl and $F'l_1$ are called the moments of the forces about the fulcrum A. The term "moment" is a useful one; for, suppose we wish to estimate the effect of a weight attached to a horizontal lever-arm at a distance of l_1, = two feet, from the fulcrum A, and again at a distance of three feet. We know by experiment that the effect of these weights, placed at these distances, is proportional to the weight, and also to the distances that the weight is placed from the fulcrum. Our standard for estimating the effects of the weight applied at different distances must evi-

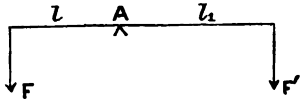

FIG. 52.

dently be some pull or force applied at the opposite side of the fulcrum at our unit distance, whatever we decide that this unit distance shall be.

Suppose that we call this unit distance one foot. We shall, therefore, have the unit force F at the distance of one foot, balancing some other pull or weight at some other distance on the opposite side of the fulcrum. Let this second pull or weight be F', and let it be placed at the distance l_1, we shall have $F \times 1 = F' \times l$; or, our standard force at the distance of l is equal to the product of another balancing force at a distance of l_1. In order, therefore, to find the effect of the application of a pull or a weight at different distances from a fulcrum, in terms of a known force or pull at a distance of one from the same fulcrum, but on the opposite side, we must make use of the product of the pull or weight by its distance from the fulcrum. This product is called, for convenience, the moment of such a pull or force.

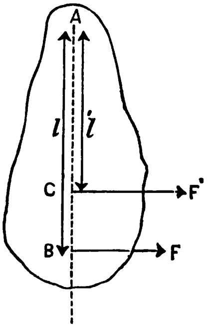

FIG. 53.

Suppose that the compound pendulum A B (Fig. 53) is struck by a force F, at the point B, at a distance of l from the fulcrum A, it is evident that the moment $F\,l$ would be equal to some standard force applied at a distance of unity from the fulcrum A. If the pendulum should be struck at C, at a distance of A C from A, in order to communicate the same swing to the pendulum that the blow at B does, this force, F^1, multiplied by its arm l^1, must be equal to the force F multiplied by its arm l, since

they are both compared by the same force multiplied by the arm unity.

Hence we have $F^1 l^1 = F l$, or $F^1 = \frac{F l}{l^1}$.

EXPERIMENT 80.—Suspend a small lead ball by a silk thread of a suitable length (Fig. 54). Place a comparatively stiff piece of steel spring, one end of which is confined in a vise, directly beneath the vertical pendulum. Draw the unconfined end of the flexible steel out of the vertical through a certain angle, attaching a thread to it and to a point, *A*. On burning the thread at *A*, the steel will strike the weight of the pendulum, and urge the latter up a height, *h*. The work the steel does in striking the pendulum-weight *W* is measured by the weight lifted through the height *h*, or $M g h = W h$. The force with which the piece of steel strikes the weight *W* is the same as that with which the weight *W*, on its fall down the arc or down the height *h*, would strike the steel spring when it is unbent, and in the same vertical line as the weight *W* when it is at the lowest point of its swing. The velocity with which the bit of steel strikes *W* is also approximately the same as would be acquired by the weight *W* in falling down the height *h*. We say approximately, for the resistance of the air and the friction of the point of support of the pendulum use up a portion of the energy of the steel spring.

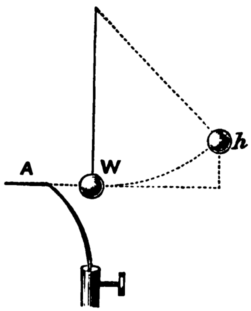

FIG. 54.

In order to find the force with which a bit of

steel strikes a blow, and compare the force with the lengths of the piece of steel employed, using the apparatus just described, we must measure the height h. This can be done by placing a piece of paper beside the pendulum, so that the height to which the pendulum-bob ascends can be noted. The work done in lifting the pendulum to the height h is then—

$$W h = m g \frac{v^2}{2 g}, \text{ or } F = M v.$$

Knowing h we can obtain v, the velocity, and also F.

Compare the force with which pieces of steel of different lengths will strike the pendulum. This apparatus in a modified form is called a ballistic pendulum, and is used to find the velocity and also the force with which cannon-balls strike any obstacle.

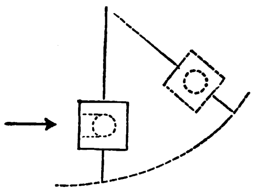

FIG. 55.

Suppose that a pendulum is made so that, when the ball strikes it, the ball should become imbedded in it, as in Fig. 55. The energy of the ball is consumed in lifting the weight of the ball and that of the pendulum-bob through a certain distance against the pull of gravitation.

In these examples it is evident that we should know what the length of the simple pendulum is which is equivalent to the compound pendulum, which, from the necessity of strength and solidity, is generally employed instead of a simple pendulum. Thus it would be difficult to make a simple pendulum which would receive the great blow

of a cannon-ball. The pendulum for this purpose must be a solid structure in which the weight is not all collected at the end of the suspending arm.

Let us now return for a moment to the case of a lever. We know that a certain weight, F, at the end of an arm, l, will balance another weight, F' at the end of an arm, l_1, or $F l = F' l_1$ (Fig. 52). Now, suppose that the lever moves up and down in a vertical plane, the space through which the force F moves is the arc $a a'$ (Fig. 56), and the space through which F' moves is $b b'$. If

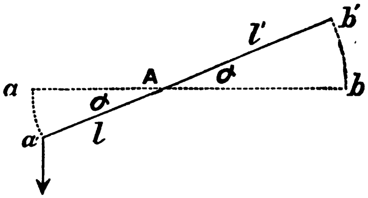

FIG. 56.

the angle corresponding to these arcs is α, we shall have $l \alpha = a a'$, and $l_1 \alpha = b b'$, for we can prove by actual measurement, with a pair of dividers and a string, that in any case $\frac{l\alpha}{l_1\alpha} = \frac{a a'}{b b'}$; and the value of the force $F = \frac{F' l_1}{l}$.

Let us suppose that the lever $a b'$ moves in a horizontal plane, to obtain the work done by the force F in moving through the arc $a a^1$, we must multiply the force by the space passed through, or the arc $a a^1$, and, to obtain the equivalent work done by the force F' in moving through the arc $b b'$, we must multiply the force F' by this arc. Hence,

$$F l \alpha = \frac{F' l_1 l_1 \alpha}{l} = \frac{F' l_1^2 \alpha}{l}.$$

By comparing the work done by different

weights or forces attached to the lever and moving through different arcs, we find—

$$Fla = \text{work} = \frac{F' l_1{}^2 a}{l} = \frac{F'' l_{11}{}^2 a}{l} = \frac{F''' l_{111}{}^2 a}{l}, \text{ etc.,}$$

or $F' l_1^2 = F'' l_{11}{}^2$, etc.,

or the different expressions for the work done are proportional to the products of the different forces by the squares of the length of the arms. These products are called "moments of inertia." They enable us to estimate the effort or work that is required to move a body through a certain arc or a certain distance with definite angular acceleration.

Let us illustrate the use of the term "moment of inertia" by the tilt-hammer.

This apparatus is often employed in iron-mills to give great blows to masses of heated metal. It con-

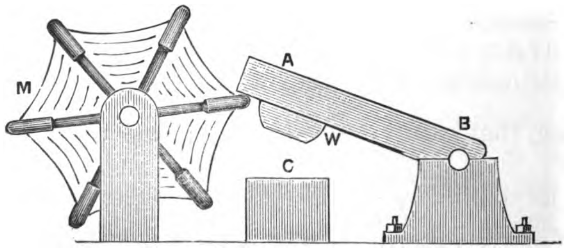

FIG. 57.

sists of a heavy weight (Fig. 57), W, at the extremity of a lever, AB; regular blows are given at one extremity of this lever by means of a revolving wheel, M. The work that the weight W does is proportional to F', the force exerted at C, multiplied by the square of the arm l', or the distance of the weight from the fulcrum B. If the mass of the weight at C is m, we have $F^1 = Mg =$ weight at C. Hence work

done by the fall of this weight through the angle a is equal to the energy which was used to force the end A through the same angle but over a different arc, or $F l a = \text{work} = \frac{F^1 l_1^2 a}{l} = \frac{M g l_1^2 a}{l}$.

Here we see that the product $M l_1^2$ occurs; this, as we have said, is called the moment of inertia, and we shall use the letter K to denote it in future. If the weight of the trip-hammer is not all collected at its extremity—and it never is in practice—it is evident that every particle of the arm of the trip-hammer obeys the same law as the weight of the simple trip-hammer which we have been using as an illustration. If we call such a particle m, and its distance from the fulcrum x, we shall find that the total work done by the fall of the trip-hammer will be the sum of the work done by each particle falling through its own arc. We should find that, in summing up the work of each particle, the whole work, or

$$F l a = \frac{mg x^2 a + m_{,} g x_{,}^2 a + m_2 g x_{,,}^2 a, \text{ etc.,}}{l}$$

$$= \frac{M g r^2 a}{l} = \frac{K g a}{l}.$$

(If M represents the whole mass and r the radius of gyration, or that length whose square is the mean of all the squares of the distances of the particles of the body from the axis.) In general the moments of inertia are calculated. A table of these for simple bodies will be found in the Appendix.

Masses will therefore have the same effect upon the conditions of motion of a revolving body, if they possess equal moments of inertia, and we can therefore replace one mass by another at a suitable distance, without altering the conditions of motion.

EXPERIMENT 81.—Obtain two brass rods of equal length (Fig. 58), one of which screws into the other; suspend this compound rod by a wire, one end of which is screwed between the rods A and B. On twisting the rod through a small angle in the horizontal plane, it will swing to and fro under the influence of the torsion of the suspension wire, and the apparatus constitutes what is called a torsion pendulum. On the extremities of the horizontal bar of this pendulum, slip equal hollow cylinders of wood or metal. If the mass of one of these cylinders is M, and its distance from the axis of suspension is b, its moment of inertia will be Mb^2. Remove these weights and slip on two others, each of mass M^1. If these are placed so that

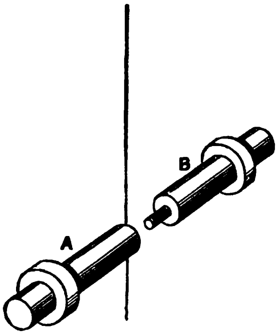

FIG. 58.

$$Mb^2 = M_{\prime}b_{\prime}^{2}, \text{ or } b_{\prime}^{2} = \frac{Mb^2}{M_{\prime}},$$

the time of swing will not be altered.

Verify the law, and also ascertain the laws of the torsion-pendulum, by altering the length of the suspension, and changing the weights on the horizontal rod.

Experiment 81 enables us to reduce a weight from one point to another, so as not to alter the conditions of motion of any revolving body. Let us take the case of the wheel and axle. Suppose that the weight P acts on the wheel with the arm

AB, and the weight Q on the axle with the arm CD (Fig. 59). Suppose, also, that the strings, by means of which these weights act, are perfectly flexible. If the moment of the weight P is greater than that of Q, the weight P will descend. The weight Q, acting with its arm DC or b, is equivalent to a force $\frac{Qb}{AB} = \frac{Qb}{a}$ acting at B, which is opposite to the force due to the weight P. Hence

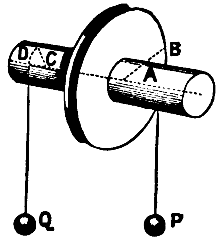

FIG. 59.

we have a resultant force $P - \frac{Qb}{a}$ acting at B. The mass $\frac{Q}{g}$ is reduced from $\frac{Q}{g}$ to $\frac{Qb^2}{ga^2}$ by removing it from the distance b to that of a. Hence the mass moved by $P - \frac{Qb}{a}$ is $M = \left(P + \frac{Qb^2}{a^2}\right) \div g$.

If the moment of inertia of the wheel and axle $= \frac{Gy^2}{g}$, its inert mass reduced to $B = \frac{Gy^2}{ga^2}$, and we then have

$$M = \left(P + \frac{Qb^2}{a^2} + \frac{Gy^2}{a^2}\right) \div g$$
$$\frac{Pa^2 + Qb^2 + Gy^2}{ga^2}.$$

Or the acceleration of the weight P

$$= \frac{\text{moving force}}{\text{mass}} = \left(\frac{P - \frac{Qb}{a}}{Pa^2 + Qb^2 + Gy^2}\right) g a^2.$$
$$= \left(\frac{Pa - Qb}{Pa + Qb^2 + Gy^2}\right) g a.*$$

* See Weisbach's "Mechanics"—wheel and axle.

Prove this last equation by the method employed in Experiment 67.

At a distance z from the axis of rotation let us place a mass, M, whose effect is the same as the sum of all the particles multiplied by the square of their distances from the axis, or $\Sigma m r^2$. We then have, Experiment 81, $M z^2 = \Sigma m r^2$, or $M = \frac{\Sigma m r^2}{z^2}$. Suppose the force P acts upon the mass M. This will impart an acceleration $g' = \frac{P}{M} = \frac{P z^2}{\Sigma m r^2}$. The time of a simple pendulum of the length z, under the acceleration g', is $t = \pi \sqrt{\frac{z}{g'}}$. Hence, $t = \pi \sqrt{\frac{z \Sigma m r^2}{P z^2}}$ $= \pi \sqrt{\frac{\Sigma m r^2}{P z}} = \pi \sqrt{\frac{K}{D}}$.

This discussion is necessary for the comprehension of the magnetic pendulum, and its use in measuring the work done by a magnet in its swing. The pendulum, which acts under the influence of gravitation, serves to give us our estimate of time, and also of the earth's gravitation-force. In the same way the swinging magnet can give us a standard of time in terms of the earth's magnetic force; and also, knowing this time, we can obtain the magnetic force of the earth, and the work a magnetic pendulum does in swinging under the influence of this force.

We have seen that a magnet, which is suspended horizontally by its middle, is a horizontal pendulum which moves under the influence of the magnetic force of the earth. It is also a compound pendulum; for we can not put all the magnetic attracting matter at one place, at a definite distance from the point of suspension. Each particle of magnetic

matter μ acts at a certain arm, l. Instead of the expression $T = \pi \sqrt{\frac{K}{m\, l_1 g}}$, which gives the time of a compound pendulum vibrating under the influence of the directing force g of the earth, we shall have $t = \pi \sqrt{\frac{K}{\mu\, l\, T}}$, in which μ is the attracting mass of the magnet, $\mu\, l$ its moment, T the directive force of the earth's magnetism, which now takes the place of the earth's gravitation-force. Taking the square of t, we have $t^2 = \frac{\pi^2 K}{\mu\, l\, T}$, or $T = \frac{\pi^2 K}{\mu\, l\, t^2}$. If, therefore, we knew the product $\mu\, l$, we could obtain the earth's magnetic force. Methods will be given later to determine the value of $\mu\, l$. The importance of the measurement of T will be understood, when it is seen that all the measurements in electro-magnetism depend upon a knowledge of the value of this force—just as all measurements of work and force in mechanical units depend upon our knowledge of g, or the earth's gravitation-force.

For the present, therefore, we will content ourselves with obtaining the value of $\mu\, l\, T = M\, T$ (by putting $\mu\, l = M$, in which M represents the sum of $\mu\, l$ for every particle of the magnet) for different magnets.

EXPERIMENT 82.—Obtain the value of $M\, T$ for several cylindrical magnets of the same section, but of different lengths.

The value of K can be calculated from the length of the magnet and its diameter expressed in centimetres, and its weight in grammes. (See table of moments of inertia in Appendix.)

In obtaining the time of swing, the magnets should not be allowed to swing through large arcs, and they should be suspended by long strands of silk, which are freed from torsion. This can be accomplished by unwinding a silk thread, and taking one of the strands and moistening it slightly between the fingers, in order to straighten it.

CHAPTER VIII.

MEASUREMENT OF MAGNETIC FORCE.

LET us now obtain an idea of the magnitude of the attracting forces with which we deal in physics. It is not difficult to obtain a conception of the amount of the earth's gravitation-pull. A pound-weight exerts what we call a certain force. It will break a strand of silk by which it is suspended. It will cause a certain deflection on a spring-balance. Its constant pull on a falling body will give it a velocity of about thirty-two feet at the end of the first second of its fall, and will make it pass over sixteen feet during the first second. The earth's pull is exercised upon every particle in a body, and the whole pull, therefore, increases with the mass of the body. The effect, therefore, of the fall of a ton-weight through sixteen feet would be very great. Turning our attention, however, to the attracting force between a magnet and the earth, we find that the force is very feeble compared with that of the gravitation-pull of the earth.

EXPERIMENT 83.—Magnetize an ordinary knitting-needle, by stroking the north pole of a strong magnet upon one half, and the south pole of the same magnet along the other half. Repeat this half

a dozen times. The knitting-needle will then be found to be a magnet. Endeavor to balance it upon a knife-edge at or near its middle. It will be found to be impossible. The earth, therefore, attracts one end (making it dip), and repels the other end. Slip a very small piece of cork along the magnet to its middle; run a very fine needle through this piece of cork, as near as possible to the knitting-needle, and at right angles to it (Fig. 60). It will be found that the needle dips from the horizon through an angle of about 74° (in latitude of Cambridge, Massachusetts). It tends, therefore, to put itself along a line of magnetic force; and these lines, therefore, make this angle with the horizon. The accompanying diagram shows that the earth's magnetic force can be resolved into a horizontal force, H, and a vertical force, V. The horizontal magnetic pendulum, which we have considered, moves under the influence of the horizontal directing pull H. It will be seen from the figure that the vertical pull of the earth's magnetism is much greater than the horizontal force, H. To obtain an idea of the amount of the pull of this vertical force, attach a very fine thread to the upper end of the knitting-needle N, as shown in Fig. 60, and ascertain how much weight must be attached to the thread to bring the needle into a horizontal position. When it is in this horizontal position, the repelling force of the vertical component of the earth's mag-

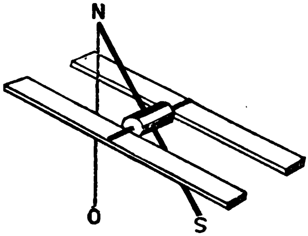

FIG. 60.

netism on *N* is balanced by the downward pull of the attached weight, or the earth's gravitation. It will be found that a very small weight will bring the dipping needle into a horizonal position. Knowing the angle between the direction of the dipping needle and the horizontal (74°), and having found the pull of the vertical component *V*, in terms of the earth's gravitation, we can construct the parallelogram of forces, and find the value of *H* in terms also of the earth's gravitation (Fig. 61).

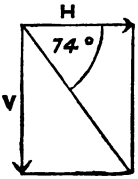

FIG. 61.

EXPERIMENT 84.—It will be found that the readiest way to bring a dipping needle into a horizontal position, or to deflect a compass-needle through a certain angle, is to bring another magnet near it. We can not conveniently compare the gravitation pull of the earth with either the vertical magnetic pull or the horizontal magnetic pull. It is necessary to employ magnets to ascertain the value of the magnetic attracting force.

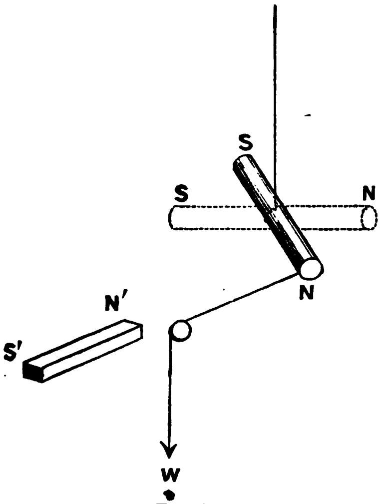

FIG. 62.

We find by our previous experiment that we can not measure the magnetic pull which a magnet

$N^1 S^1$ exerts upon another, NS, as in Fig. 62, by attaching a thread to S, and passing it over a pulley and attaching a weight, W, to it. If the magnet $S^1 N^1$ is a feeble one, we can counteract the effect of $S^1 N^1$ upon $S N$, by putting another magnet so that its pull upon $S N$ may be opposed to that of $S^1 N^1$. We can then estimate all magnetic pulls and pushes in terms of the effect of this last standard magnet. It is found, however, that no magnet remains the same for any length of time, and our magnetic standard would vary from time to time. It would be difficult, moreover, as we have seen by the previous experiment, to compare our magnetic standard with the earth's gravitation-pull, which is our invariable standard of force and of work. We must, therefore, find some method of expressing the earth's magnetic pull in the units which are based upon our standards of time and of force, and which we employ in mechanics to estimate force and work.

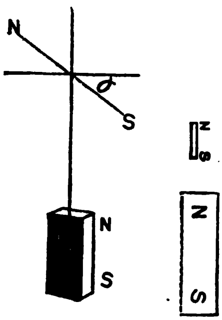

FIG. 63.

The effect of one magnet on another can be seen in two ways. Suspend a magnet by a single fiber, without torsion, in front of another magnet, and set it to swinging (Fig. 63). It will be found that the number of vibrations is determined by the strength and distance of the larger magnet, $N S$. When $N S$ is placed directly north of the small suspended magnet $n\, s$, and near it, its effect upon the small magnetic pendulum is much greater than that of the

earth, as can be seen by comparing the number of vibrations of ns before and after NS is placed before it. One effect, therefore, of one magnet upon another can be detected and measured by the magnetic pendulum.

The time of this little pendulum is $t = \pi \sqrt{\frac{K}{\mu l T_1}}$, in which T_1 is the strength of the magnetic pull upon the small suspended magnet, and, in the case just considered, is the sum of the pull of the earth and of the large magnet NS. Without the large magnet the little pendulum swings under the influence of the earth's magnetic pull alone.

Now, place the large magnet directly to the east of the small suspended magnet ns. Its effect is to deflect the small magnet ns through an angle, α. There are two effects, therefore, that a stationary magnet, NS, can exert upon a movable or suspended magnet—one is to increase the number of its vibrations, and the other to deflect it out of the magnetic meridian. We have already obtained the value of the time of swing of a suspended magnet. Let us now see if we can estimate the quantitative effect of one magnet upon another.

Suspend a small magnet about one centimetre long, made from a bit of watch-spring, and provided with a long, light pointer, by means of a silk fiber without torsion. Place a paper graduated circle, or two paper protractors suitably joined, beneath the magnet, so that the center of the graduated circle shall be immediately beneath the middle of the suspended magnet.

Instead of providing a magnet with an index or pointer, which moves over a graduated circle, we

can place a little mirror upon the magnet, and reflect a spot of light upon a strip of paper which is graduated into millimetres and centimetres. When the magnet moves, the beam of light is reflected along the scale. In this case we have a long index-arm without weight. In Fig. 64, *B* represents a box in which a little magnet, *n s*, provided with a small, plane mirror (a piece of thin looking-glass

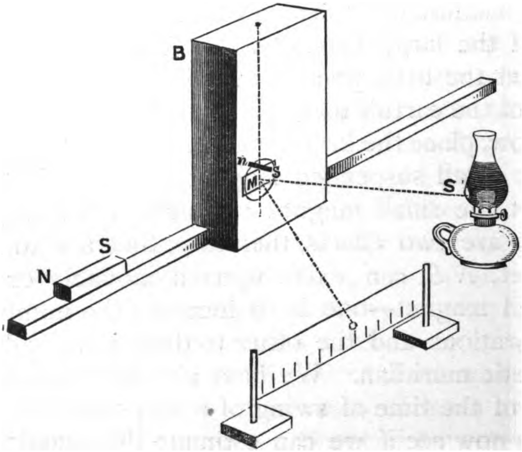

Fig. 64.

will answer) is suspended, the magnet is placed at right angles to the mirror. *S* is a small clear spot in a blackened lamp-chimney. The light after reflection from the mirror *m* is received upon a strip of ground glass or oiled paper, upon which a suitable scale is placed. A paper scale can be pasted upon the ground glass, or a wooden scale can be suitably placed against the glass. The glass allows the spot of light to be seen through

the glass by the eye, which is placed in front of the scale.

It is necessary to place a lens of long focus in front of the plane mirror *M* (unless one uses a concave mirror, see Appendix), in order to obtain a defined image of the slit *S* upon the ground glass. In order to understand this, arrange in a dark room a lens, *l*, and a lamp with a slit, *s* (Fig. 65), so that an image of the slit *S* may be formed by the lens at *F*. Then place a piece of looking-glass behind the lens, and ascertain the position *F′* of the reflected image. It is not necessary that the suspended magnet should be placed in a dark room: for, in a room only moderately darkened, the eye when placed in line with the reflected beam can perceive an image of the slit upon the ground glass or oiled paper. It will be found necessary to obtain the deflection of the little suspended magnet in terms of the tangent of the angle through which it is deflected by any magnet *N S*, placed to the east or west of it. In the first place, arrange the ground glass with its scale so that it shall be parallel to the mirror. This can be done by setting the magnet in motion, and by moving the scale until the deflections first on one side of the zero-line and then on the other are equal. When

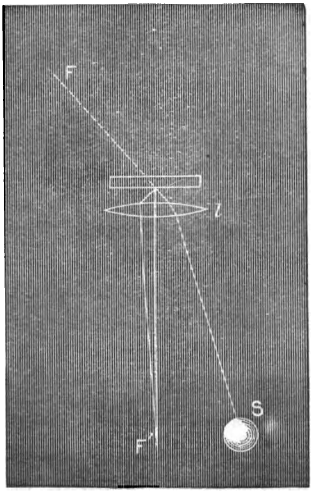

FIG. 65.

the mirror turns through an angle, α, the angle θ (the tangent of which is n, or the number of scale deflections divided by d, the distance of the mirror from the scale) is twice the angle α. This can readily be proved by setting up a piece of looking-glass on a protractor, and observing, by means of a telescope or eye placed at I, the reflections of the scale from this mirror when it is turned through a small angle.

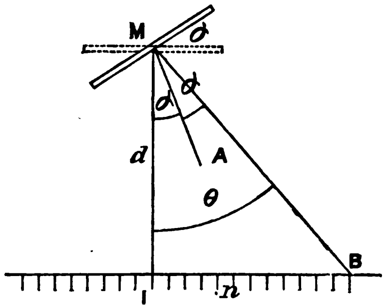

FIG. 66.

Since, however, IM and MA are perpendiculars, or normals to the mirror in its two positions, the angles between these normals is the same as the angle between the two positions of the mirrors. Moreover, a beam of light passing along BM, it is known, will make equal angles with a normal, MA, to the reflecting surface.* Hence the angle $\theta = 2\alpha$. The tangent of $\theta =$ tangent of 2α,

$$= \frac{\text{scale reading}}{\text{distance of scale to mirror}} = \frac{n}{d}.$$

For very small angles, $\tan \alpha = \frac{1}{2} \tan 2\alpha = \frac{1}{2}\frac{n}{d}$.

If a higher degree of accuracy is desired, the value of $\tan \alpha$ in terms of $\tan 2\alpha$ can be obtained from trigonometrical tables.

With this apparatus let us now ascertain how the force of attraction between magnets varies.

* Prove this experimentally.

With the object of finally obtaining a measure of this force in gravitation units, place a comparatively large magnet, which is five centimetres long and one centimetre in diameter, directly to the east of the suspended magnet, and at a distance of at least ten centimetres.

If this latter magnet is very strong, or if it is placed very near the suspended magnet, the latter will point directly toward it, in the east and west line in which the large magnet is placed. The further the large magnet is removed from the suspended magnet, the less does the suspended magnet point toward the stationary or larger magnet. This is because the magnetism of the earth exerts its effect; and the suspended magnet comes to rest at a certain angle, a, from the north and south line, under the influence of the two pulls. Why does it come to rest at a certain angle? In order to answer this question, try the following experiment: Weight a rod heavily at one end (Fig. 67). Suspend the rod vertically by slipping it through a hole at A, upon a smooth peg, so that the weights and the connecting rod can readily revolve about A. Attach a spring-balance to the lower weight and pull it out horizontally through a vertical angle of a. The product of the pull in pounds expressed by the spring-balance and the projection of the half length

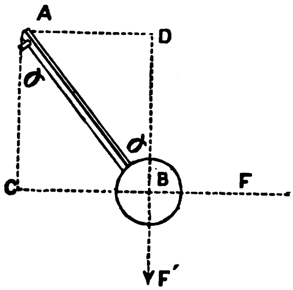

FIG. 67.

$A B = l$ of the rod on the vertical, or $A C$, will be found to be equal to the product of the weight at B expressed in pounds multiplied by the projection of $A B$ on the horizontal, or $A D$. Now we have shown that the projection $A C = A B \cos \alpha$, and the projection $A D = A B \sin \alpha$.

Hence, if we call the horizontal pull on $B = F$, and the vertical pull of the earth on $B = F'$, we shall have $F A B \cos \alpha = F' A B \sin \alpha$, or $F = \frac{F' \sin \alpha}{\cos \alpha} = F' \tan \alpha$.

Returning now to our magnetic forces, we find that the pull F in our last example on the spring-balance is replaced by the magnetic pull F, and the earth's gravitation pull F' by the earth's magnetic pull $F' = M T$.

Hence, we shall have here also $F = F' \tan \alpha = M T \tan \alpha$. We place $F' = M T$, for the earth's pull is proportional to its own magnetic effect, T, and to the strength of magnetism, M, of the sus-

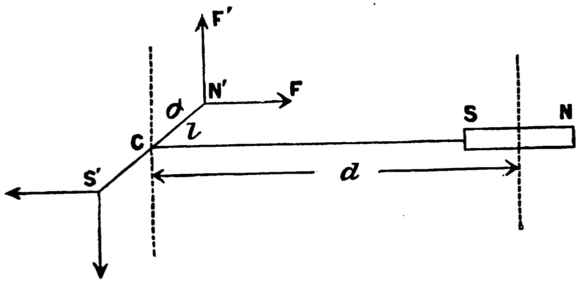

FIG. 68.

pended magnet, and to nothing else. We find, also, by placing the magnet $N S$ (Fig. 68) at differ-

ent distances to the east of the movable magnet, *N′ S′*, that the tangent of the angle varies in a certain way with the distance. It also varies as the strength of the suspended magnet and as the strength of the large magnet, *N S*.

Prove that the force *F* varies inversely as the cube of the distances from the center of the magnet *NS* to the center, *C*, of the suspended magnet, or that $\frac{F}{F_0} = \frac{M\,T \tan \alpha}{M\,T \tan \alpha_1} = \frac{\tan \alpha}{\tan \alpha_1} = \frac{\frac{1}{d^3}}{\frac{1}{d_1^3}} = \frac{d_1^3}{d^3}$. Since the force also varies as the strength $M M_1$ of the two magnets, we shall have $\frac{M M_1}{d^3} = M\,T \tan \alpha$ for any distance, *d*, of the magnet M_1 or *NS* from the magnet *M* or *ns*. From this we find that $\frac{M_1}{T} = d^3 \tan \alpha$.

We shall now give another method of ascertaining the relation between *F* and *F′*, with the purpose not only of determining this relation, but also of illustrating the effect of couples in mechanics.

Experiment 85.—A small bolt (Fig. 69, *A*) has a nut and a washer. Between the nut and the washer a long pointer, *P*, is confined. This pointer can move over a graduated circle or the arc of a protractor when the bolt is turned in its support. To the lower end of the bolt is fixed a small cross-arm, *B*, to which a small magnet, *N S*, can be attached by two strands of silk, *S S*, free from torsion. Fix the suspension-head, *B A*, in the top of a suitable box, with a plate of glass so that the position of

rest of the magnet can be observed. For this purpose the suspended magnet should be provided with a little bristle or index which should point to a suitable mark on a bit of mirror beneath it when the suspension-threads are parallel and in the same vertical plane. Place a large magnet outside the box and east or west of the suspended magnet. It will then be found necessary to turn the suspension-head through a certain number of degrees in order to bring the index of the suspended magnet back to its zero-mark, or to its resting-place when the outer magnet is removed.

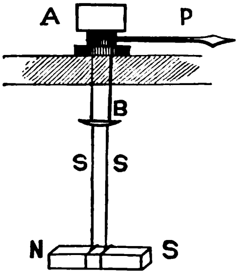

FIG. 69.

A body hung up by a bifilar suspension constitutes a pendulum. Its time of swing can be obtained by the methods we have given; or this time can be calculated from the formula $t = \pi\sqrt{\frac{K}{D}}$, in which K is the moment of inertia of the suspended body, and D is the directive force of the suspension. The tension on the suspending fibers caused by the weight of the suspended body produces a couple, which tends to bring the suspended body back to its position of rest. The bifilar suspension, therefore, constitutes a horizontal pendulum. Suppose that A B (Fig. 70) represents a magnet which is suspended by two fibers, A C and B D. (For the sake of clearness the suspension apparatus and magnet are much

enlarged.) When another magnet, NS, is brought near the suspended magnet, it will turn about a vertical axis, $E\,E'$. In order to keep it in the

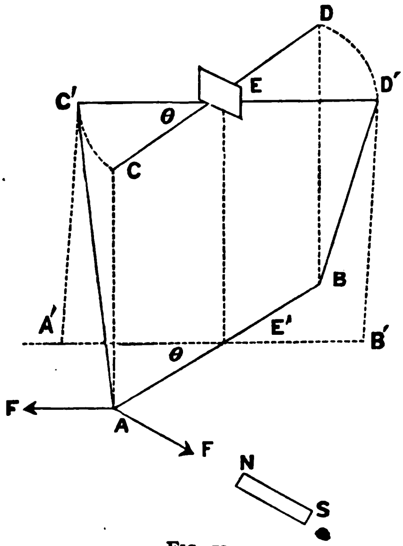

FIG. 70.

original position, $A\,B$, we must turn the suspension head, $C\,D$, through a certain angle, θ. If, now, the stationary magnet $N\,S$ is removed, the suspended magnet $A\,B$ will move under the action of the couple $D \sin \theta$ (in which D is the directive force, or $F'\,l$), into the plane $A'\,C'\,D'\,B'$, and will vibrate from one side to the other of this plane until it finally comes to rest. In the position $A\,B$ we should therefore have the couple due to the mutual action of $N\,S$ and $A\,B$, or $F\,l = D \sin \theta$, and for another position of the magnet $N\,S$ along the same line perpendicular to $A\,B$, $F'\,l = D \sin \theta'$, or

$$\frac{F}{F'} = \frac{\sin\theta}{\sin\theta'} = \frac{d_1^3}{d^3} \text{ (see Experiment 84).}$$

Instead of using a graduated circle with an index, or pointer, a little plane mirror can be placed at E, with a small lens in front of it, as in Experiment 84. In this case we shall obtain the tangent of $2\,\theta$, and from a table of natural sines and tangents (see Appendix) we can easily obtain $\sin\theta$. In this experiment the stationary magnet must be much larger than the suspended magnet, and must be placed at a considerable distance from the latter. The suspended magnet should not be more than two centimetres in length, and the stationary magnet NS should be at least fifteen centimetres distant.

EXPERIMENT 86.—To find the value of the earth's magnetic force, T, or the horizontal component of the earth's magnetism, suspend the magnet which we have used in deflecting the suspended magnet (Experiment 85) in a position where there are no other magnets or masses of iron, and determine the time of its swing under the influence of the earth's magnetism. Then calculate its moment of inertia, measuring its length and diameter by non-metallic or brass measuring instruments. In this way we shall obtain the time $t = \pi\sqrt{\frac{K}{MT}}$, or $MT = \frac{\pi^2 K}{t^2}$ (1). Now place the same magnet about thirty centimetres to the east or west of a small suspended magnet which we used in the previous experiment, and obtain the value $\frac{M}{T} = d^3 \tan a$ (2). Then divide eq. (1) by eq. (2), and we shall obtain $T^2 = \frac{\pi^2 K}{t^2 d^3 \tan a}$,

or $T = \frac{\pi}{t}\sqrt{\frac{K}{d^3 \tan a}}$. The measurement of $\frac{M}{T}$ should be made at equal distances to the east and west of the suspended magnet, and the mean of the distance of the points where the center of *NS* is placed should be taken, on account of the difficulty of measuring from either to the suspended magnet. The mean of the angles of deflection should also be taken. The measurements of length should be in centimetres, the weight in grammes, and the time in seconds. The value of *T* for Cambridge, Massachusetts, in 1884, was .172 C. G. S. It varies, of course, with the latitude of the place, and varies slightly also for the same latitude from year to year.

We have proved by experiment that the force of attraction between two magnets varies as the product of the strength M and M^1 of the two magnets, and inversely as the cube of the distance between their centers. It is difficult to ascertain by direct experiment how the force of attraction varies between two poles—a north and south pole, for instance—for we can never separate the two poles of a magnet. The effect of a magnet, *NS*, in the last experiment upon a small suspended magnet, *ns*, is due to the combined effect of both poles of *NS* upon both poles of *ns*. We have found that this combined effect is proportional to the strength of the magnets *NS* and *ns*, and to the inverse cube of the distance between their centers. This effect must be due to both the north pole *N* and the south pole *S* of the magnet *NS*. What, therefore, must be the effect of the north pole *N* alone upon the south pole of the suspended magnet, and the effect of the south pole *S* alone upon the same pole *s*?

Suppose we assume that the mutual effect of these poles is inversely as the square of the distance between them (in Fig. 68). Calling $2l$ the length of the magnet NS, and d the distance of the center of NS from the pole N^1 of $N^1 S^1$, we shall have the force of attraction between S and $N^1 = F = \frac{\mu\mu^1}{(d-l)^2}$ (calling μ and μ^1 the magnetic strengths of N^1 and S).

In the same way we shall have the force between the pole N and the pole N^1, $F_1 = \frac{\mu\mu^1}{(d+l)^2}$. This force is in the opposite direction to F. Hence, the resultant pull between the two magnets must be the difference between F and F_1; the first, or F, being a stronger pull than the push of F_1, since S is nearer N^1 than N.

The difference $F - F_1 = \frac{\mu\mu_1}{(d-l)^2} - \frac{\mu\mu_1}{(d+l)^2}$
$$= \mu\mu_1\left(\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right) = \frac{2M\mu_1}{d^3}.*$$

Hence we see that the force of attraction between a north pole and a south pole really varies inversely as the square of the distance between them, since by assuming this law we are led to the same result as is given by direct experiment.

This is the same law as that of gravitation. The sun and the earth attract each other directly as the

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$$* \mu\mu_1\left(\frac{1}{(d-l)^2} - \frac{1}{(d+l_1)^2}\right) = \frac{4dl\mu\mu_1}{(d^2-l^2)^2}$$
$$= \frac{4dl\mu\mu_1}{d^4\left(1-\frac{l^2}{d^2}\right)^2} = \frac{4l\mu\mu_1}{d^3} = \frac{2M\mu_1}{d^3},$$

reducing to a common denominator, neglecting the square of l, and placing $2l\mu = M$.

product of their masses, and inversely as the square of the distance between them. There is this marked difference, however, between the attraction of gravitation and that of magnetism, that the force of attraction of gravitation increases as the masses of the two bodies, and can be made indefinitely great. The attraction, for instance, between the earth and a steamship is enormously greater than that between a pound-weight and the earth. The force of attraction, however, between a magnet weighing half a pound and another magnet weighing half a pound is not necessarily increased when the weight of these magnets is increased. A piece of iron or steel can only retain a certain amount of what is called magnetism, or what we have termed M. It is a fallacy to suppose that, if we increase the size of a magnet indefinitely, we shall increase the force of attraction which it exerts upon another magnet.

We now see, if we could place one pole of a magnet free from the other, in a uniform magnetic field, that it would move along the lines of uniform magnetic force, just as a falling body moves along the lines of uniform gravitative force. Such a magnet would move with an accelerated velocity, and would strike with a certain force. Its velocity would be (if it fell vertically along lines of magnetic force, of strength T) $v = \sqrt{2\,(M\,T + g)\,h}$, instead of $v = \sqrt{2\,g\,h}$.

A small being, like a Lilliputian, placed in a magnetic field and surrounded by particles of matter, all of which possessed one magnetic pole, would obtain the same laws for falling bodies that we ob-

tain upon the earth. A magnetic body, however, placed in a uniform magnetic field, always possesses two poles. The law of the fall of such bodies along lines of magnetic force must be modified, since one pole is attracted while the other is repelled.

We have supposed that the movements of our magnetic pendulums have taken place in a uniform magnetic field—that is, a space in which all the lines of force are parallel and of the same strength. It is hard to obtain a uniform magnetic field. We exist, however, at the same place on the earth's surface in a uniform gravitation field in which the lines of force are all sensibly vertical and equal. It must be remembered that our investigation of the law of bodies falling under the influence of gravitation takes place very near to the surface of the earth. If we should represent the earth by a circle one foot in diameter, the highest tower now built would not be appreciable if we should attempt to draw it on the circumference of such a circle. Therefore, when a body falls to the earth, we consider that the same force acts upon it through the whole of its fall. This force is proportional to the mass of the stone and to the mass of the earth, and inversely proportional to the square of the distance between the center of the body and the center of the earth. This distance really varies during the fall of the body, but its variation, even if the body could fall from the top of a tower 100 feet vertically to the base, would be inappreciable compared with the distance between the top or bottom of the tower and the center of the earth. The radius of the earth is about 21,120,000 feet. It will be seen that 100 feet, or even 1,000 feet, is very

small compared with a distance of 21,120,000 feet. The other manifestations of forces of attraction which we commonly see are due to electricity.

EXPERIMENT 87.—Hang up by a fine silk fiber a pith-ball, and, having rubbed a piece of sealing-wax upon the coat-sleeve, present it to the pith-ball: it is immediately attracted to the wax, and after receiving a charge from the wax it is quickly repelled. We find by experience that bodies charged by what is called the same kind of electricity—that is, two pith-balls electrified in the same manner by a piece of sealing-wax—will repel each other.

We also know that we can induce an opposite charge on a non-electrified body by bringing it near an electrified body, for the non-electrified body will be attracted to the charged body. The body *A* (Fig. 71) charged with positive electricity will attract a negative charge to the end *d*, and repel a positive charge to the end *c* of a neighboring conductor *c d*. This phenomenon is called electrostatic induction.

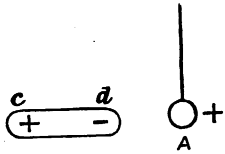

FIG. 71.

A thunder-cloud charged positively will attract a negative charge beneath it on the earth, and repel a positive charge to the other side of the earth. If we should coat the inside of a tumbler with tin-foil (Fig. 72), and also the outside, keeping the coatings separate, we should find that the positive charge on the inner coating will induce a negative charge on the inside surface of the outer coating of tin-foil,

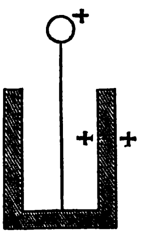

FIG. 72.

and repel a positive charge to the outside surface of the outer coating. By connecting the outer coating with the ground, the positive charge passes into the ground, and we have left a negative charge held on the outer tin-foil by a positive charge on the inner tin-foil. The two electrifications are then said to be bound, and the tumbler constitutes what is called a Leyden-jar. By such an arrangement we can therefore obtain a definite charge of electricity.

It is important to obtain a clear idea of electrical attraction and of its magnitude compared with the attraction of gravitation and of magnetism. We shall, therefore, refer the student to general treatises on the various phenomena of electro-statics, and confine ourselves to the fundamental experiments which are necessary in order to obtain a clear conception of the relation between the various forms of energy.

EXPERIMENT 88.—Coat the exterior of a smooth, large tumbler, or similar glass vessel (Fig. 73), with tin-foil to within two inches of its rim. Fill it to within this distance of the top with strong sulphuric acid. The acid absorbs moisture and also answers for the inner coating of a Leyden-jar. It will be found that this vessel thus filled can be charged with electricity by connecting the tin-foil with the ground and passing sparks from an electrical machine into the sulphuric acid. A special contrivance, by means of which this can be done conveniently, will be described later. Suspend from a torsion-head similar to that in Fig. 69, but one much smaller, and made with more delicacy, a light, flat disk of paper, covered with tin-foil, and balanced at the end of a bright metallic pointer, which is connected

with the tin-foil on the flat disk. The suspension should consist of two fibers of silk—the finest or ultimate fibers of a white silk thread (cobwebs can be used). These two fibers should be a small distance apart. Suspend from immediately beneath the point of attachment of these silk threads a very fine platinum wire, with a small platinum or lead weight,

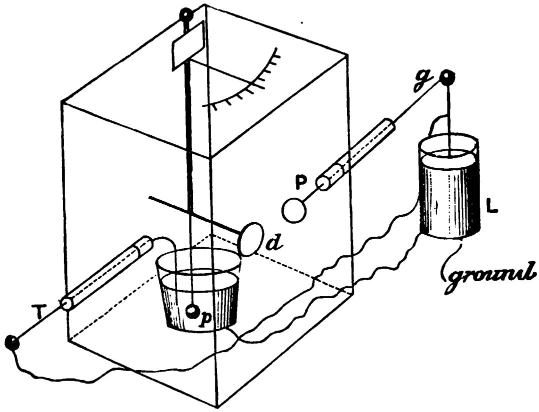

FIG. 73.

p, at the bottom. Let this disk, with its index, hang vertically over the tumbler, and the platinum weight and wire be immersed one or two inches in the sulphuric acid. When the sulphuric acid is electrified as we have described, the disk will also be electrified by the same kind of electricity, but will not move unless some body is brought near it. The torsion-head should be placed in the top of a box, the inside of which should be coated with tin-foil, with the exception of one of the vertical sides, which should be of glass. A metallic ball, stuck by

a bit of wax to the end of a hollow glass rod, *Pg*, and connected through the center of this glass rod by a bit of wire to another metallic ball at *g*, can be electrified at *g*. The attraction or repulsion between the electrified ball *P* and the disk *d* will require that the suspension-head should be turned through a certain angle in order to maintain the disk at zero. By placing the electrified body *P* at different distances from the position of rest of the disk *d*, it will be found that the suspension-head must be turned through different degrees which are proportional to the force between the disk *d* and the electrified body *P*, and also inversely proportional to the square of the distance between *d* and *P*. It is convenient to electrify the sulphuric acid of the tumbler by passing a fine platinum wire or rod through a hollow glass tube, *T*, which is passed through the side of the box. The glass rods serve to insulate the charging-rods from the box. The tumbler should be placed upon a metallic plate, and this plate should be connected with the ground.

By connecting the inner coating of a Leyden-jar, *L*, with the rod *Pg*, and also with the charging-rod *T*, we shall have approximately the same charge upon *d* as upon *P*, and we can therefore measure the repulsion between two similarly electrified bodies, by means of the method of Experiment 85, the electrical attraction taking the place of the force *F* of magnetic attraction. In this case, ascertain the relation between the sines of the angles through which we must turn the suspension-head in order to balance the couples due to the attracting body *P* at the different distances of *P* from *d*. The interior of the box should be per-

fectly dry, and the charging-rod T, and the charged body P, should be carefully insulated from the box by means of glass tubes which are frequently cleaned with alcohol. It is best to use a plane mirror upon the suspension-head, with a stationary lens, and a ground-glass scale, according to the method described in Experiment 84.

By a method similar to the one we have just described, it has been proved that the force of electrical attraction varies inversely as the square of the distance between the electrified bodies and directly as the product of the charge of electricity upon the bodies.

Here we have another example of attracting force, which varies inversely as the square of the distance between the bodies. The bodies which are attracted move along lines of electrical attracting force. These lines are not straight, in general, between two attracting pith-balls. By electrifying two plane metallic disks, we can obtain a uniform field of attracting force at the center of these disks. At the edges, however, the lines of force are curved. A small being, placed between the two disks, if he could measure the laws of falling bodies, would find laws similar to those we have discovered in the case of the attracting force of gravitation (if all the matter about him were electrified by the same kind of electrification). Electricity differs from magnetism in this respect. We can charge a pith-ball or a piece of steel with one kind of electricity, and thus observe the effect of, we will say, a positive charge upon another positive or a negative charge, whereas we can not separate one magnetic pole from another and opposite one. On the

other hand, electricity resembles magnetism in this, that a positive charge, *A* (Fig. 71), induces two states—a negative and a positive—in all neighboring bodies. A positive magnetic pole, or a north pole, induces a south and a north pole in all surrounding magnetic matter. In the latter respect, electricity and magnetism are closely analogous, while they both differ from the effect of gravitation upon different masses.

We do not know whether electricity or magnetism is some peculiarity of matter—some grouping of the particles—some quick or slow action of molecules with respect to other particles or molecules. It is probable that we shall never know what electricity is, or what magnetism is. Indeed, there is as much mystery in regard to the force of gravitation as in regard to electrical and magnetic force. What we can do, however, is to observe and measure the relations of electrical and magnetic force to gravitation force, and to study, as we shall see, the great law called the conservation of energy, which connects all the manifestations of force. We see from the preceding examples how to make measurements of the attracting force of gravitation, of electricity, and of magnetism. There is still another manifestation of electricity, closely allied to that of magnetism, by which attractive force is shown.

EXPERIMENT 89.—Having formed a spool by gluing to the ends of a hollow cylinder of wood—five centimetres long, with an external diameter of two centimetres, and an internal diameter one and a half centimetres—two circles of wood, in order to form the ends; wind upon this spool from four to

six layers of No. 18 insulated copper wire. Connect the ends of this spool or coil of wire with the poles of a Daniell cell, and place the coil to the east of a small suspended magnet. It will be found that one end of the coil acts like the north pole of a magnet upon the suspended magnet, while the other end acts like the south pole. We could, therefore, use a coil of wire through which a current of electricity passes as a magnet. Place a cylinder of iron in the center of the spool, making the length of the iron nearly equal to that of the spool. It will be found that the effect of the coil is now greatly increased. Repeat Experiment 84, replacing the stationary magnet by the coil provided with an iron core. This coil is called an electro-magnet. It is therefore evident that we can make a strong and uniform magnetic field, or field of force, by means of a current of electricity which is made to pass through a coil of wire. This field exists without the use of an iron core. The latter, however, strengthens it greatly.

EXPERIMENT 90.—Support the electro-magnetic coil vertically, and, having placed a sheet of stiff paper upon one end, sprinkle or sift some fine iron-filings upon this paper. On tapping the paper the filings will be seen to arrange themselves in lines, exactly similar to those seen when a permanent steel magnet replaces the coil. These magnetic figures can be obtained when a coil is used without a magnetic core, especially if a strong current is used. Place two electro-magnetic coils vertically face to face, so that the north end of one is opposite the south end of the other. Slip a piece of card-board between these poles, not placing the poles

more than two inches apart. On sifting iron-filings upon this card-board they will be seen to arrange themselves in approximately straight lines between the poles. The space between the poles is a field of force, and all our previous remarks and experiments in regard to fields of attractive force apply also to this field of force.

In order to make measurements of force, it is essential that the field of force should be uniform—that is, the lines of force should be straight, and the pull or push along them should be the same at any point in the field. If the lines of force are not straight, a pendulum in swinging across these curved lines of force would not be uniformly acted upon by equal parallel forces. It would move under the influence of one force in one direction, and at another time under the influence of another in a different direction. There could be no uniformity in its motion in different parts of such a curved field of force. In all our experiments, therefore, we endeavor to make our measurements in fields of uniform force, whether these fields be gravitation fields, magnetic, or electrical fields. We generally think of the force of gravitation as a force which is always the same in the same place, and as always acting along a line directed to the center of the earth. Since the center of the earth is so remote, the lines of gravitative force are considered parallel, and their divergence from parallelism is so slight that it can not be detected in a laboratory. Suppose, however, that a meteor should be falling in outer space somewhere between the earth and the sun. It would evidently move in a curved line of force under the effect of the gravitating force of

the sun and of the earth. All the meteors near each other in space would move approximately along the same curved lines of force. The field of force, however, would not be uniform, but would vary at different points along these curved lines. It is only when the meteors are close to the surface of the earth that they move along straight lines of force, and in a uniform field of force, for then the influence of the sun becomes inappreciable.

We have no difficulty in obtaining uniform fields of gravitative force, for we are like the minutest speck of magnetic matter infinitely near a magnetic pole: the moment that we become of a size comparable with the attracting mass, or find ourselves between different attracting masses, at a considerable distance from each, we cease to move in uniform fields of force. In electricity and magnetism the little horizontal pendulums which we use to measure the direction and the amount of the attracting force are comparatively large bodies, and are not insensible, compared with the size of the attracting magnets or the electrified bodies whose attracting force we wish to measure. Hence, we must arrange our electro-magnets or our magnets and electrified surfaces so that there shall be a uniform field of attracting force where we swing our little measuring pendulums.

We have seen that a current of electricity in passing through a wire exerts an attracting effect upon a neighboring magnet. Let us examine this new attraction still further.

EXPERIMENT 91.—Place a wire immediately over or under a compass, so that this wire will run north and south. Pass a current of electricity through

this wire (Fig. 74). It will be seen that the needle of the compass will make an angle with this wire. There is, therefore, a force of repulsion at A acting upon the poles of the magnet, repelling it from the wire. By changing the poles of the battery, so that the current may flow through the wire W in opposite directions, it will be found that the magnetic needle changes the direction of its deflections. It will be observed that the needle is only deflected through a certain angle, and that this angle varies with the strength of the current sent through W. We know that the earth's magnetism acts upon each pole of the needle in the north and south line along which the wire W runs. The deflection of the needle is due to the earth's magnetic pull, $M\,T$ (proportional to M, the strength of the little magnet which is used, and to the earth's magnetism, T), and to the repulsive force, F, due to the current in the wire W.

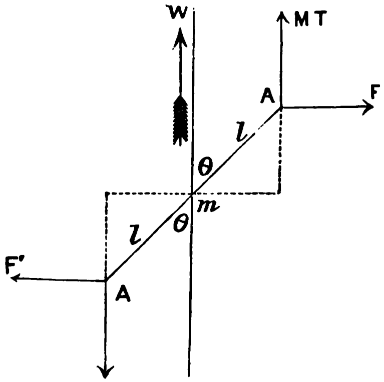

FIG. 74.

Thus we see that the force F of the current on the magnetic pole of the needle multiplied by the projection of half the length of the needle on the wire, or $l \cos \theta$, l being the half-length of the needle, is equal to the earth's pull, $M\,T$, multiplied by the

projection of the half-length of the needle on the horizontal line perpendicular to the wire.

By lifting the wire, through which the current of electricity passes, vertically above the needle (Fig. 75), still keeping it parallel to its first north and south direction, we find that the needle is still deflected, but through smaller and smaller angles. The force from the current through the wire is still perpendicular to a vertical line passing through the center of the magnet; that is, the force F acting upon the magnet $A\ A$, tending to turn it out of the plane $m\,n\,n'\,m'$, is perpendicular to the line $m\,n$, or to its parallel $m'\,n'$.

If, therefore, we could have a long strip of steel which had but one north pole, this strip of steel would wind itself like a serpent around a wire which conveyed an electrical current; for, since the force of the current acts everywhere perpendicular to the line joining any part of the magnetic pole and the nearest portion of the wire conveying the current, the mechanical effect would be to whirl the magnetic pole around the current.

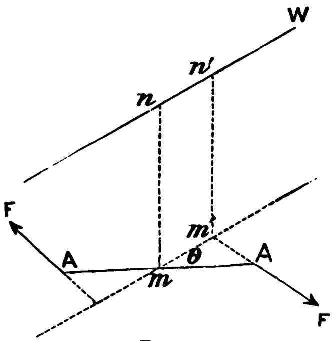

FIG. 75.

By means of a circle of wire through which a current passes, and so arranged that the needle placed at the center shall always move in a uniform field of force, we can evidently measure the strength of an electrical current. For the force, or push, or

pull which the current exerts upon a needle is evidently proportional to the strength of the current which passes through the circle of wire and to the radius of this wire; for the stronger the current the larger the deflection of the magnetic needle, and the smaller the radius of the circle the larger the deflection. Every portion of the circle carrying the current exerts an electro-magnetic effect. The length of the circumference of a circle is $2\pi a$; the whole electro-magnetic effect is, therefore, proportional to $2\pi a S$, where S is strength of current passing through the wire, and to the strength of the magnetic pole, m, and inversely proportional to the square of the radius of the circle of wire. Hence we have $F = \frac{2\pi a S m}{a^2}$.

This force acts with the arm $A\,m \cos\theta$ (Fig. 74), and balances the earth's magnetic effect $M\,T$ acting with the arm $A\,m \sin\theta$. Hence we have

$$\frac{2\pi a S m}{a^2} A\,m \cos\theta = M\,T A\,m \sin\theta, \text{ or } S = \frac{T a}{2\pi} \tan\theta.$$

If there are n turns of wire instead of one turn, we shall have $S = \frac{T a}{2\pi n} \tan\theta$.

Since we can measure T (see Experiment 86), we shall have the strength of the current expressed in terms of distances or lengths, of moments of inertia and of weight. Hence it is but a step to express the strength of an electrical current in terms of mechanical work. Before doing this, let us gain clearer ideas of work.

CHAPTER IX.

POTENTIAL AND WORK.

We have seen that a body falling under the influence of gravitation will do an amount of work which depends upon the height fallen through. The greater the height, the greater the amount of work that can be obtained from a falling body. In the case of gravitation, we know that the falling body descends along vertical lines of force under the influence of the force which acts toward the center of the earth. In order to lift a body through a height, we do work constantly against this force of gravitation. In lifting a body, and in thus doing work against the force of gravitation, we know that at any moment we can let the body fall, and thus regain by its blow the energy we exert in lifting. The moment that we have let the body fall and it has exerted its effort in driving a pile or in molding a mass of hot iron, we can get no more work out of it unless we expend work upon it by lifting it again to a height above the point where it is to strike.

Our ideas of work done and of work that can be done by falling bodies depend, therefore, upon the height we lift the body. How can we compare the energy which can be obtained from two weights by

letting them fall upon a certain place? Suppose one of the weights weighs two pounds, and is at a height of ten feet above the point we wish to strike; suppose another weight of four pounds is ten feet below the same place: it is evident that we must raise the latter weight above the level of the point we desire to strike, in order to get the same work from it that we can get from the two-pound weight by simply letting it fall.

The work of the two-pound weight can be called plus work, and the work that has to be expended on the four-pound weight to simply lift it to the height of the point where the two-pound weight strikes can be called negative work. If we keep on lifting the four-pound weight we know that we can get back the work we do by letting it fall to the point where the two-pound weight strikes, but we can not get back at the point *A* which we wish to strike the negative work we have done on the four-pound weight in lifting it simply to *A*. The height, therefore, of a body above or below the point at which we wish to do work is an important consideration in our estimate of the work necessary to be done in lifting bodies to a certain height and letting them fall upon some intermediate point, *A*. From the whole positive work we shall have to subtract the negative work in order to estimate the expense or the loss of energy. If bodies are being lifted to various heights from various depths, and allowed to fall to some intermediate point, *A*, and this operation is going on at different points on the earth's surface, there would be great confusion in the note-books of the different observers unless some uniform level should be assumed, to which all the heights could

be referred. The level of the sea is commonly taken as the level to which we refer all our heights. Since most of our operations are confined to points above this level, the convenience of such a plane of reference as the sea-level in all our estimates of heights, and therefore of the work done against and in the direction of gravitation, can be illustrated by the operation of leveling. This consists in obtaining the difference in height between two places in order to see, for instance, if water can be made to flow from one point to another. The leveling instrument (Fig. 76) consists merely of a telescope which is provided with a long spirit-level, L, which is placed parallel to the axis of the telescope. This telescope can turn around horizontally on a vertical axis. It is placed on a tripod directly over some mark, A, and adjusted so that the line of sight CD through the telescope shall be horizontal. A vertical graduated rod, BM, is placed at some point, B, and the height C above B, where the line of horizontal sight strikes the rod, is read. The rod BM is then moved forward a measured distance to B', and the

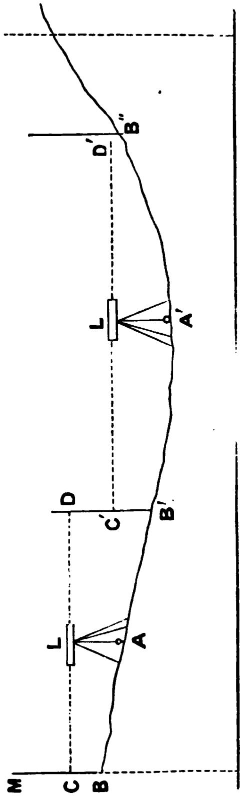

Fig. 76.

intersection of the same horizontal line of sight with the vertical rod is found at D. Then the leveling-instrument is carried forward to another position, A', and what is called a back-sight is taken at C' upon the vertical rod placed at B', and a fore-sight upon the vertical rod when it is carried to B'', and so on. The height or depth of the point B'' above or below B can be found by comparing the different readings upon the vertical rod. It is evident that we should have to register in our note-books plus heights or heights above B, and negative heights or heights below B, in order to decipher our notes. We should have great difficulty unless we called certain heights which were above B + heights, and those that were below B — heights. In order to prevent confusion, it is customary to take an arbitrary plane twenty or more feet below the lowest depression that can possibly occur in going from B to B''. Suppose the distance of this arbitrary reference plane is sixty feet below C, if we simply subtract from sixty feet the difference of the back-sight and the fore-sight for the point B', we shall have the height of C', and, knowing B' C', the height of the contour-line $B\ A\ B'\ A'\ B''$ at B' above this reference plane, and our result will always be positive. It is evident that this plane can be taken at any depth so long as it is below the lowest valley that is likely to occur. The height of B'' can be obtained, knowing that of C', and so on.

EXPERIMENT 92.—Lay off on paper the following line of country between B and B'' from this page of an engineer's note-book:

Distance of reference plane below point B, forty feet; distance between the stations, one hundred feet.

Station.	Back-sights.	Station.	Fore-sights.
B	5 feet.	B	2 feet.
B_{i}	7 "	B_{i}	10 "
B_{ii}	1 "	B_{ii}	5 "
B_{iii}	1 "	B_{iii}	15 "
B_{iv}	2 "	B_{iv}	2 "

In the same way the work that is necessary to be expended in rolling a weight over the line of country represented by this contour-line, can best be estimated by referring all the heights and valleys to some reference plane beneath the lowest valley.

Let us now consider the work we do in moving a body about so as to keep it constantly at the same height above a reference plane. It is evident that, since we do not lift it against the force of gravitation, we do no work against this pull. Since the weight is not allowed to fall along a line of gravitating force, it does no work in the direction of the line of gravitation. The work of the moving weight, therefore, with respect to gravitation, is nothing. There is work done upon the body in moving it against friction from place to place; but this work is estimated by the horizontal pull upon the body, and the distance through which this pull must be exercised. It has nothing to do with the height of the plane along which the body is moved above the plane of reference or the sea-level, except that the friction of the body depends upon the pressure it exerts upon the plane along which it is moved, and this pressure depends upon the force of gravitation. As long as the force of gravitation remains sensibly the same, it does not matter how high the plane is along which a train of cars moves; whereas, the

force with which a weight falls to the level of the sea, and therefore the work that is done, varies greatly with every foot in increase of height.

If we should cut a mountain by successive planes (Fig. 77), all of which are parallel to the sea, we see from the preceding that a train of cars passing around the mountain along any line cut by any one plane, *A B*, would do no work with respect to gravitation; for it would always move at the same level above the sea-level. If it should fall from the level *A B* to the level *A′ B′*, it would do a certain amount of work by its fall, which would depend upon the

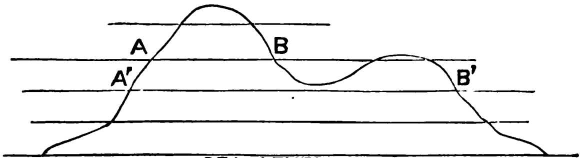

FIG. 77.

difference of level of *A B* and $A^1 B^1$. These planes and their bounding lines around the mountain are called with respect to gravitation equipotential planes and equipotential lines.

EXPERIMENT 93.—Draw the equipotential lines or contour-lines represented by the different heights of the corners of the squares (Fig. 78) formed by intersecting lines of level which have been taken by an engineer. It is evident that one of these contour-lines of height 2 will run from 2 on line *C D* in the manner shown in Fig. 78. This curved line will represent the boundary of a horizontal section of the country, *A B C D*, taken at the level 2. Draw in this way the other contour-lines, then represent the

section along a line of railway, *EF*. The sides of the little squares may be taken as one hundred feet.

The term equipotential is of great significance in physics. If we should measure the work done by a body in falling to the level of the sea from different heights, we could designate these heights and the planes passing through them as points where the potential has a certain value, or the planes as equipotential planes.

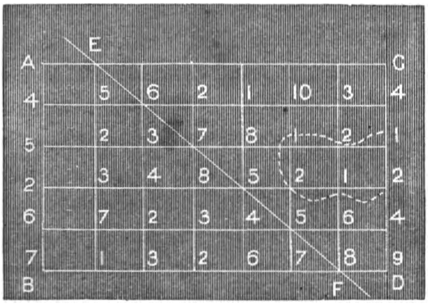

FIG. 78.

When a body strikes a blow after having fallen from a certain height, we call the energy with which it strikes kinetic energy: it is energy that results from the motion or fall of the body. Before the body fell, it had, by virtue of its position, or height above the point struck, a certain advantage. It could strike a certain blow if it fell. It has what we call potential energy, or energy of position. This energy it sacrifices in falling. It gains kinetic energy but loses potential energy, so that we have the sum of the potential energy and the kinetic energy always equal to a constant. Let us lift a certain weight vertically above the sea-level. We are doing work against the force of gravitation; on the other hand, we are gaining potential energy. Let us now let the body slip from our grasp and fall a

certain distance. During its fall it is losing potential energy, but is constantly gaining kinetic energy. Therefore, with reference to the sea-level, the falling body loses potential energy but gains kinetic energy. When it strikes at some point above the sea-level, it has manifested kinetic energy which can be used in driving a pile or striking any blow, but it has lost potential energy with respect to the sea-level. We have gained the energy of the blow, but we have lost our advantage of position. Suppose we have an elastic string (Fig. 79) attached horizontally at A and B; if we pluck this by the middle and allow it to swing back, it is evident that a particle at m will be losing potential energy with respect to the line of level $A\,B$, but it will be gaining kinetic energy which will be proportional to the distance $m\,m_1$. The potential energy at m_1 will be only proportional to the height $m_2\,m_1$, whereas before it was proportional to the height $m\,m_2$. The bob of a pendulum loses potential energy on one side of its swing, and gains kinetic energy. On the opposite side it loses kinetic energy and gains potential energy.

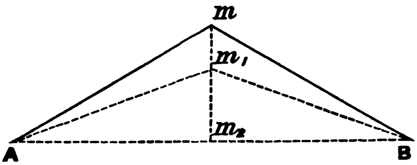

FIG. 79.

The remarks we have made upon the expressions equipotential lines, and planes, or surfaces, with reference to gravitation, apply equally well to the lines and surfaces in magnetic and electrical regions. It is evident that a pendulum will swing

in the same time at all points on the same equipotential plane, for the force of gravitation is the same at every point on this equipotential plane. We could therefore determine different contour-lines around a high mountain, cut out by imaginary equipotential planes, by swinging a pendulum at different points and passing the planes through these points; these planes would then cut out contour-lines or equipotential lines along which the force of gravitation would be the same. This method of determining equipotential planes is difficult to carry out on the surface of the earth, for the force of gravitation is sensibly constant for considerable variations in height. No difference in the length of a seconds-pendulum would be detected (except by very delicate experiments) between its length at the sea-level and its length at the height of a mountain one thousand feet above the sea. We can, however, use this method in electricity and magnetism. Let *S* (Fig. 80) be a strong electro-magnet, with a large pole or extended surface, *a b*. The lines of force will be approximately straight across the section *a b;* move a little magnet, *N S*, or compass, along a line, *m n*, so that its vibra-

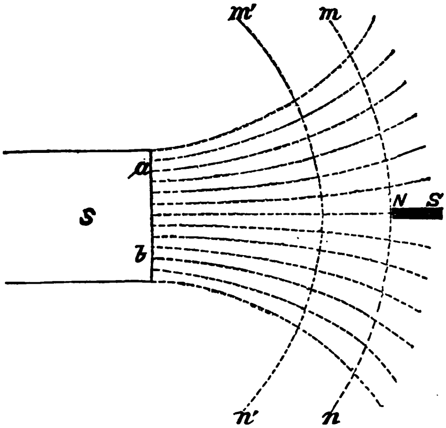

FIG. 80.

tions will be the same at every part of *m n*. This line *m n* will be perpendicular to the lines of force emanating from *a b*, and will be an equipotential line. There will be no tendency of the magnet to move along the equipotential line *m n*. The only way that one magnetic pole could do work under the influence of the attracting pole *S*, is to move under the influence of the attracting pole from one equipotential line *m n* to another, *m' n'*. It will do work in moving from one equipotential line to another, just as a body under the influence of the force of gravitation will do work in falling from one equipotential line or plane to another. We have, therefore, magnetic potential energy as well as gravitative potential energy, and it is evident that we can replace the magnetic pole *S* in the above example by an electrified body, and the small magnet *N S* by another electrified movable body, and find that we could arrive at the conception of electrical equipotential lines and surfaces and electrical potential energy, just as we measure potential energy due to gravitation by reference to a plane or a zero equipotential plane above which we must lift a body in order to get any work out of it. In all that precedes we have assumed that the equipotential surfaces are concentric spherical surfaces. If they are not, the force at each point is not necessarily the same. In magnetism and electricity we can, however, arrange any suitable field of force.

In experiments, however, upon the attractive or repulsive force of electricity, our results are generally much influenced by the proximity of neighboring objects; for it is well known that an electrical charge upon any body will induce a similar charge upon all objects in the neighborhood. The earth's

atmosphere, moreover, contains a certain varying electrical charge. There are electrical fields of varying strength all around us, and we are forced to adopt some reference plane by which we can estimate the work necessary to be done in lifting bodies against an electrical attraction up to this plane, or the work that can be gained by letting bodies fall to this reference plane. We take the electrification of the ground as zero; but the earth may have a certain charge and the moon another. We know that the electrical charge of the surface of the earth is constant, as far as our senses can perceive. We can, therefore, take the equipotential surface of the earth as our plane of reference in electricity, just as we take the level of the sea as our plane of reference in regard to heights on the surface of the earth, and to the potential energy at different heights.

We speak, therefore, of a certain electrically charged body, A, as having a certain potential with respect to another body. We mean that we must do a certain amount of work to remove it beyond the influence of the second body, and this amount of work is measured by reference to the potential of the earth.

The best illustration of the use of the potential of the earth, as a reference for all electrical potentials, is in the action of an ordinary cell of an electrical battery with reference to the earth.

EXPERIMENT 94.—To prove that in any galvanic cell one metal or pole has one charge of electricity, or plus charge, and the other an opposite or negative charge. The instrument we have used in Experiment 88 can be modified so as to show this. The movable index is replaced by an aluminum needle, shaped as is shown in Fig. 81.

Above the needle, and rigidly connected with it by a piece of aluminum, is a light concave mirror, *m*. A platinum wire is attached to the needle which dips into the sulphuric acid of the vessel, *V*, as before. This needle can be electrified by electrifying the sulphuric acid. When it is placed in a dark room, a beam of light from a point in the soot-blackened chimney of a kerosene-lamp can be thrown upon the mirror, and its reflection can be received upon a piece of ground glass placed at the focus of the mirror, as is shown in Fig. 72.

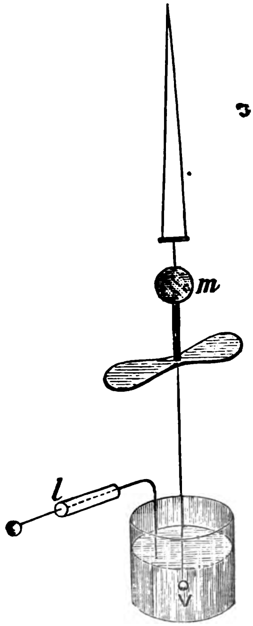

FIG. 81.

This arrangement will be found to be very sensitive, and will show the nature of an electrical charge. The poles of a battery might be presented one after the other, at a certain distance from this needle, and their different electrical effects observed. But the electricity on the hand or clothes of the observer would be likely to mask any effect that the poles of the battery might have. The interior of the box, therefore, which protects the needle from currents of air, is coated with tin-foil, and is connected with the outside of the little glass tumbler or vessel *V* (Fig. 82), which is also coated on the outside with tin-foil. This coating of tin-foil is

then connected with the ground, in order that it shall all be at the potential or charge of the earth, and shall remain constant, since it then becomes a part of the earth. Four hollow quadrants of brass, one of which can be pushed in and out, are then placed upon glass pillars, and the aluminum needle is hung at the center of the circle which these quadrants form.

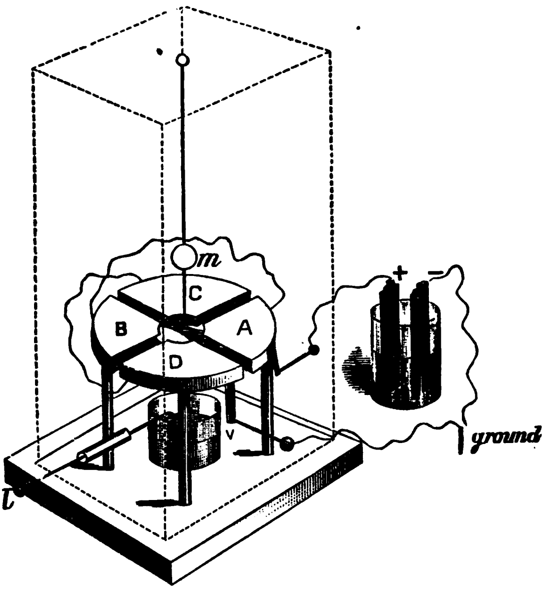

FIG. 82.

The quadrant A is then connected by a fine platinum wire with B, and the quadrant C in the same way with D. The charge which A receives will then be communicated to B, and also any which D receives will be shared by C. Connect at first all the quadrants together and with the ground by means

of the metallic rods, which run from two adjacent quadrants beneath them, and are carefully kept from touching the tin-foil or any part of the inclosing box. By thus connecting the quadrants with the ground they can all be kept at the constant potential of the earth, and the needle will not move, since it is equally attracted on all sides. Connect now one set of quadrants with the ground and with one pole of a battery, and the other set with the other pole of the battery, as is shown in the accompanying figure. It will be found that the spot of light will move in one direction; by reversing the poles of the battery, or, in other words, connecting different poles with the same set of quadrants, it will be found that the needle will move in the opposite direction. Thus it will be seen that the poles of a battery have opposite charges, and are, therefore, at different potentials with respect to that of the earth.

Compare the difference of the charge on the poles of one battery and that on two which are connected, the positive pole of one with the negative pole of the other; also, when the positive of one is connected with the positive of the other.

There is thus a difference of potential between the poles of a battery; or, in other words, the charge upon one of the metals or poles of a battery-cell is below that of the earth. It is in the condition of a weight upon which work must be done in order to lift it to the earth's electrical level. It is in a position analogous to a weight which is below the level of the sea. The charge upon the other pole has a potential above that of the earth. It is in a condition analogous to a weight above the level of the sea, and it can do work by falling to the electri-

fication of the earth. If we, therefore, connect both poles with the ground we can conceive of the electrification of the earth flowing to the negative pole and the electrification of the positive pole flowing to the lesser charge of the earth. This happens in the case of a cell when the circuit is completed through the ground. We expect, from our analogy in the case of mechanical work, that work must be done by this rise and fall of electrical potential; for, wherever there is a difference of potential, work can be done. When there is no difference of potential, then we have the case of two heights standing at the sea-level, and no work can be obtained from them with reference to this level. In the case of the cell we do not know why there should be this difference of potential between two different metals. It is not necessary at present to enter into theories in regard to the cause of the difference of potential which produces what is called electro-motive force. All we wish to know at present is, how we can connect the phenomena of energy, and measure them all by the same units of time, space, and mass. We can, however, think of a battery as an engine which is pumping up water through one pipe and delivering it through another. Suppose that the engine is in-

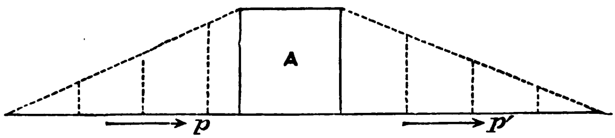

FIG. 83.

closed at *A* (Fig. 83), and that the water is lifted through one pipe *p* and discharged through another,

p'. Suppose that we can not study the mechanism of the engine at A. All we can see and measure is the difference and direction of the pressure in the two pipes p and p'. This difference of pressure is analogous to the difference of potential between the poles of an electrical battery or cell, which maintains in some unknown way an electro-motive force along the wire. We do not know whether there is any current in a wire like that produced by the flow of water in a pipe. We only know that there is a difference of level, and that there are conditions for the performance of work in an electrical circuit. If we could measure this work by the same units with which we measure mechanical work, or the work that is done, for instance, by a weight falling a certain distance under the effect of gravitation, we could compare electrical work or electrical engines with mechanical work, or the work done by machines.

Before doing this, let us look more carefully at the way we measure work. We have seen that we can measure the work done by a body falling vertically under the influence of gravitation, or by a body falling down an inclined plane. The work of a man could be estimated by the weight he can lift above the level of the sea. The unit which is generally adopted in England and America for measuring work is a horse-power, which is taken to be equivalent to raising 33,000 pounds per minute one foot above the level of the sea. It does not matter whether it takes one minute to raise this weight or one year, the work done is the same. Five or six men are, roughly speaking, equal to one horse. What are called the mechanical powers enable us to distribute the work done in various ways. For instance, with the simple lever

(Fig. 84), by means of a small weight, W, moving through a considerable distance, $B B'$, we can move a much larger weight through a small distance, $A A'$. We see here an illustration of the conservation of energy, for the small weight at B', in falling through $B B'$, can do a certain amount of work by its blow at B. The energy of this blow, if properly redirected, could raise an equal weight to the same height from which it fell. The law of the lever is merely the statement that the energy developed by two falling bodies is proportional to the weights of those bodies, and to the distances through which they fall. These distances in the case of the lever are the sines of the angles, described by the points on the lever to which the weights are attached:

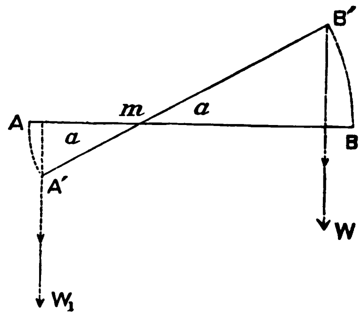

FIG. 84.

Or, $W_1 \, A' m \sin \boldsymbol{a} = W m \, B' \sin \boldsymbol{a}$ (Fig. 84).

Since sine $\boldsymbol{a}$ cancels out, we have the law of the lever: $W_1 A' m = W m B'$, or $\frac{W_1}{W} = \frac{m B'}{A' m}$. The law of the compound lever can also be shown to be a statement of the conservation of energy. A compound lever is merely a collection of simple levers, by means of which the pressure at one point is distributed always in accordance with the law that the distance through which the force or weight which is applied travels, multiplied by this pressure on

weight, must be equal to the distance through which the weight to be lifted traverses, multiplied by the weight; or, in other words, the work done by the falling body must be equal to the work done on the body which is lifted.

All forms of levers can be resolved into simple levers by an application of the law of work.

The law of the simple pulley can also be immediately deduced from the law of work. If the weight W is pulled up a certain height by the descent of W_1 (Fig. 85), we know that the work done by the fall of W_1 through its distance d, must be equal to that done on W through its rise d'.

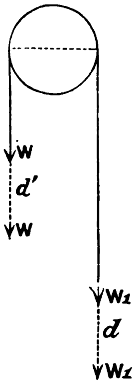

FIG. 85.

If the pull on the rope of the pulley is not vertical but inclined at an angle on opposite sides of the pulley, we know that the work done on W can be compared to that done by W_1 by comparing the vertical heights through which W rises and W_1 falls.

No matter how many pulleys we may have, the law of the combination can be deduced by ascertaining how the vertical distance through which the pressure or pull is exerted compares with that through which the weight is lifted. In Fig. 86 the pull on the rope passing around the lowest pulley, A, is $\frac{1}{2}$ that of the pull exerted by W, since it is supported one half at m, and one half by the next pulley, B. The pull on the rope passing around B is $\frac{1}{2}$ of $\frac{1}{2}$ of W for the same reason, and the pull on the rope passing around the pulley C is $\frac{1}{4}$ W. If, therefore, we wish to balance the work done by the weight P in falling a vertical

distance, h, we must allow a weight, W, to be lifted only $\frac{1}{4}$ of h.

The popular adage, that what we gain in power we lose in time, is a statement of the conservation of energy, or of the equivalence of work. We see in the case of the compound pulley that in order to lift a weight W by one $\frac{1}{4}$ as great, we must let the smaller weight pass over a space four times greater than that of the larger weight, and accordingly occupy a time in doing this four times longer than if we lifted a weight simply equal to the falling weight, or $\frac{1}{4}$ of W.

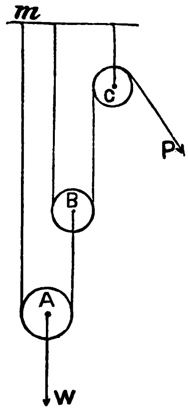

FIG. 86.

The law of the screw can be deduced from the principle of work. The force P is exerted through a space equal to the circumference of the circle, whose radius is equal to the distance of the point of application of the force from the axis of the screw. At the same time the screw is forced through the distance between two threads, and lifts a weight through this distance.

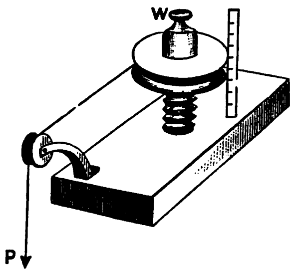

FIG. 87.

If $r =$ radius of circle around which P, the force, moves, $W =$ weight lifted, and $h =$ distance between two threads, we have $2\pi r P = W h$.

EXPERIMENT 95.—Arrange the experiment as follows: Around the wood-

en circle (Fig. 87) which forms the head of a screw pass a silk thread. Pass this thread horizontally around a smooth pulley, and attach a weight, *P*, to its free end. The tension on the thread will be equal to the weight and to the force exerted at the circumference of the screw. Place a weight upon the head of the screw, and prove the truth of the formula given for the screw.

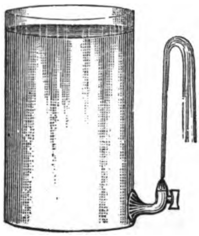

FIG. 88.

In this way, by the application of the parallelogram of forces and by the law of work, the laws of the mechanical powers can readily be obtained. By the principle of work we can also deduce the laws which govern the motion of fluids. For instance: Why should a jet of water (Fig. 88) strive to rise to the level of the reservoir which supplies it with water?

We know that any particle or drop of the water in falling from the top of the reservoir to the bottom, where the jet issues, would, by the law of falling bodies, acquire a velocity $v = \sqrt{2gh}$.

This would be sufficient, if we neglected the friction at the orifice and the resistance of the air, to carry this particle again to the height from which it fell. Hence this action of fluids is another illustration of the conservation of energy.

CHAPTER X.

WORK AND HEAT.

IT is evident that we could measure the work done by a man, a horse, or a steam-engine on any piece of mechanism, if we could compare the weight lifted and the space through which it is lifted with the force applied and the space through which it is exerted. In many cases this can be done. In most cases, however, the motion of the parts is too rapid and the spaces passed through too great for us to measure conveniently. For instance, suppose we wish to measure the work an engine does in turning an axle which revolves sixty times a minute. By grasping a slowly-revolving axle with the hand, which is protected by a thick glove, we find that we can alter the velocity of the moving axle (provided that small power is applied to it) by tightening our grasp, still allowing the axle to turn. The friction that arises between our glove and the axle is proportioned to the pressure we exert in grasping the axle.

EXPERIMENT 96.—Obtain two iron cog-wheels, one twelve inches, and the other two or three inches in diameter. Provide them with axles, and mount them as in Fig. 89. The larger cog-wheel will thus be in the position of the stone of an ordinary grind-

stone, and will gear into the smaller wheel placed at its circumference. Arrange upon the axle of the latter the apparatus represented in Fig. 89. This consists of a lever of wood, one end of which carries a balance-pan; the other is in the form of a ring cut in two, the upper half of which is hinged so that it can be lifted and clasped around an axle and then be bolted tight to this axle, as is shown in the fig-

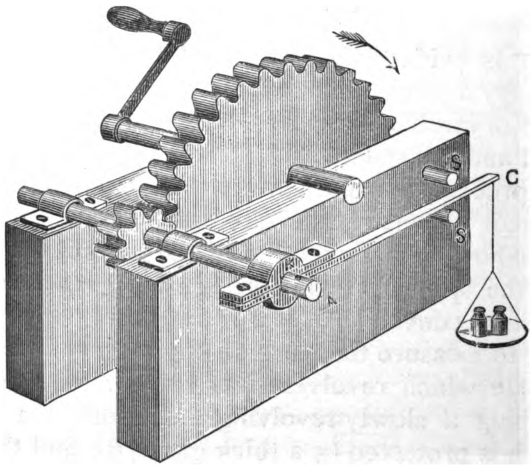

FIG. 89.

ure. When the axle is turned, it is evident that this apparatus, which is called a friction-brake, will tend to revolve with the axle. If, however, we place a suitable weight in the balance-pan, at the end of the lever, we can maintain the end of the lever in a horizontal position between the stops *S S*, when we turn the axle with a certain velocity. If we turn with a lesser or a greater velocity, the end of the lever does not remain between the two stops. When it does

remain there, we know that the weight in the balance-pan, multiplied by its length of arm, which is measured to the center of the revolving axle, is balanced by the friction which arises on the surface of the axle, between the wood and the iron of the axle, multiplied by the radius of the axle or the arm at which the friction acts. When the lever arm is horizontal, and the brake is screwed to the axis, and a certain number of revolutions of the axis is produced, with the weight W in the balance-pan, we know that the whole mechanical effect produced is expended in overcoming the friction between the shaft and the brake, and this mechanical effect is equal to the work of the revolving shaft. The friction acting in the direction of the revolution counterbalances the effect of the weight W at the end of its arm.

If we call the lever $A\ C$ of the weight a, the moment of this weight $= Wa$, and this is equal to the moment of the friction, or the friction acting with the arm of unity. If ω represents the angular velocity of the shaft, the mechanical effect $E = F\ v = W\ a\ \omega$ per second. If we call n the number of the revolutions of the shaft per minute, we shall have $\omega = \frac{2\pi n}{60} = \frac{\pi n}{30}$, and the work required $L = \frac{\pi n a}{30} W$. (This is in foot-pounds.) The weight W includes the weight of the apparatus reduced to the point of suspension of the balance-arm. To ascertain this, the apparatus is placed upon a knife-edge at K, and a string, attached to C, and to a spring-balance, will give the required weight.

In order to measure the velocity with which you

turn the large cog-wheel, hang up a pendulum, and move the handle of the axle in unison with the swings of the pendulum. This can be done by practice. Knowing the circumference of two cog-wheels, and the rate of one, we can readily determine that of the other.

A little practice will enable one to turn the axle quite uniformly once a second and twice a second, and also at some intermediate rate. In this way measure the work at different velocities.

EXPERIMENT 97.—The work done can also be measured in the following way: Attach one end of a leather band to a fixed support at *A* (Fig. 90), pass

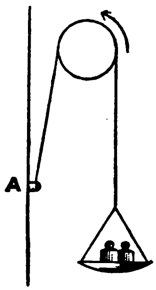

FIG. 90.

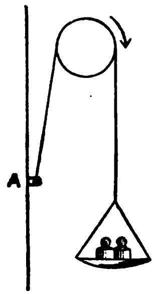

FIG. 91.

the band over the axle, and place a suitable weight in a balance-pan, which is attached to the other end of the band. Ascertain the weight that is necessary to keep the band tight when a certain velocity of rotation is given to the axle against the pull of the weight. Notice also what weight must be put in the pan to tighten the band when the axle revolves in the same direction (Fig. 91) as the pull of the weight. One experiment will then give

us $Q = P + F$, and the other $Q = P$, in which F is the friction.

By these methods, therefore, we can obtain the work that is done by transmitting power from one axle to another. If we call the work that can be done by the original machine W, and the work that can be done by the second machine W^1, the percentage or proportion of work that we can transmit will be $\frac{W^1}{W} = a$. The loss of power that is perceived is due to the slipping of the connecting belts by which the power is transmitted, and to the heating of the axles. That a part of the power is employed in heating the axles is a common fact of experience. It is a most important fact, and the whole science of modern physics is based upon it. That friction produces heat is evidenced by the heating of car-axles, by the heating of tools driven by lathes, and by the friction of an ordinary saw while it is being driven through a plank. This heating was known from the earliest times. Indeed, it is said that Indians have obtained fire by rubbing dry sticks together. No advance, however, was made in our knowledge of the relation between work and heat until an exact measurement was made of the amount of heat produced by a measured amount of work. When this measurement was made, it was discovered that mechanical work could be converted into an exact equivalent of heat, and it was further discovered that heat itself was a form of motion. We are in a condition now to make a comparatively exact measurement of work. It is only necessary for us to know how we can measure an amount of heat. Let us, first, make a comparatively rough

experiment, and afterward proceed to a more exact one.

EXPERIMENT 98.—Slip upon the axle of the smaller cog-wheel, Fig. 89, the open end of a thin brass tube (Fig. 92), having first closed the tube by a plug of plaster at a suitable point, *P*. Screw the brass tube upon the axle *A* of the smaller cog-wheel, placing leather washers between the brass tube and the screws, and also between the brass tube and the axle, to prevent conduction of heat to the axle. Drill a hole in the brass tube at *h*, sufficiently large to insert a thermometer. Fill the brass tube nearly full with water, and insert a thermometer in the water. After the water has arrived at a stationary temperature, remove the thermometer, close the little hole tightly, and put a friction-brake upon the metallic tube, having placed a suitable weight in the balance-pan, in order to keep the bar of the friction-brake in a horizontal position, while maintaining a comparatively rapid rate of rotation of the small cog-wheel and its axis. After a measured time it will be found that the water has risen in temperature. It is evident that a certain amount of heat has been absorbed by the sides of the metallic vessel, and a certain amount will be absorbed by the bulb of the thermometer when it is placed in the hot water, after the cog-wheel has ceased to turn. Knowing the work that has been expended in maintaining a certain velocity of rotation against the friction-brake, the heat that has been produced by this can be estimated in term of

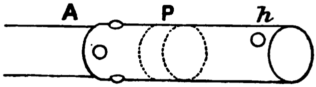

FIG. 92.

the weight of the water in pounds and the number of degrees of temperature that it has been raised. It has been found that 772 pounds lifted one foot high is the equivalent of one pound of water raised one degree Fahrenheit in temperature. You can not expect to obtain this exact number by your experiment, for it is difficult to maintain the rotation of the cog-wheel uniform. Moreover, you have neglected the amount of heat that has been absorbed by the sides of the metallic vessel and by the thermometer-bulb, and also the amount that has radiated into space while you have been making your observation. The result you will obtain, however, will be a fair approximation for the first result. In order to obtain a closer result, it is necessary to examine the methods by which we can estimate the quantity of heat. We shall then apply these more refined methods.

In the first place, let us examine our thermometer. The effect of heat upon a mercury thermometer is a matter of common experience. We know that on warming the bulb, the little thread of mercury rises in the fine capillary tube of the thermometer, and that on cooling it sinks. On surrounding the bulb of the thermometer with melting ice we find that the column of mercury remains at a fixed point in the tube of the thermometer, which is marked freezing-point (0° on the centigrade thermometer and 32° on Fahrenheit). On the other hand, if we place the bulb and tube of the thermometer in steam, carefully protecting the bulb from the bubbling water, we find that there is another stationary point, which is the heat of steam. This is marked 100° on the centigrade thermome-

ter and 212° on the Fahrenheit. These points are approximately invariable at the same place on the earth's surface. If we ascend a high mountain, we find that the boiling-point is not so high as at the level of the sea. We know, therefore, that the boiling-point is affected by the diminished or increased pressure of the column of air resting upon the boiling water. The freezing-point is also affected by differences of pressure. Moreover, the glass changes from time to time its molecular structure when submitted to changes of temperature, and these two fixed points, therefore, the freezing-point and the boiling-point, are not really invariable, though enough so for ordinary measurements. These two points, however, should be carefully examined before any thermometer is used.

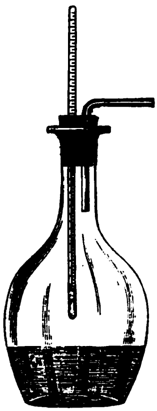

FIG. 93.

EXPERIMENT 99.—To test the freezing-point, the thermometer-bulb is placed in melting ice. The ice should be packed around the stem as far as the top of the mercury-column.

To test the boiling-point, it is necessary to hang the thermometer so that it may be entirely in the steam, and the steam should not be allowed to condense on the bulb. A simple arrangement for accomplishing this is represented in Fig. 93. Pass the thermometer through a hole in the cork, until the bulb nearly touches the surface of the water. A glass tube, also passing through the cork, and bent at an obtuse angle, allows the steam

to escape; otherwise it is evident that there would be an explosion.

After these fixed points are obtained, the space between them is divided into 100° for the centigrade thermometer and 180° for the Fahrenheit thermometer. One hundred centigrade degrees are, therefore, equal to one hundred and eighty Fahrenheit degrees, or one Fahrenheit degree is $\frac{100}{180}$, or $\frac{5}{9}$ of a centigrade degree. It will be noticed that a centigrade degree is nearly twice as large as a Fahrenheit degree. In order to reduce from the Fahrenheit scale to the centigrade, we must first subtract 32° from the reading of the Fahrenheit scale, since the freezing-point on this scale is 32° above its zero; and then take $\frac{5}{9}$ of the remainder. In dividing the space between the freezing-point and the boiling-point into equal degrees in a thermometer, it is assumed that the capillary bore of the thermometer is of the same size throughout. If it were not, it is evident that a degree at one place would be more or less than at another place. If the tube were large at one place, the mercury would move through a less space for the same increase of heat than it would if the bore were narrower. It is necessary, therefore, to carefully examine this question if great accuracy is required (see Appendix). In our present experiment the other errors of experimentation will probably be greater than the errors in the lengths of the degrees of an ordinary Fahrenheit thermometer. If the fixed points have changed, we must allow for the change. For instance, if the freezing-point is lowered, we must subtract the amount of fall from each reading.

The unit of measurement in heat is called the unit of heat.

DEFINITION.—*The unit of heat is the amount required to raise one gramme of water* 1° *C. If a gramme of water loses a unit of heat, it will fall* 1° *in temperature.*

It is necessary to prove that the unit of heat is constant for all temperatures of water—that is, it will take as much heat to raise one gramme of water from 5° to 10° as from 15° to 20°, or from 35° to 40°, etc. Otherwise our standard would change, and we should no longer have a standard unit of measurement.

EXPERIMENT 100.—Pour some water into a beaker, and, having carefully dried the outside of the beaker, place it upon a piece of wire gauze over a gas-burner (or alcohol-lamp), and heat the water to some convenient temperature—say, 30° C. Take an equal quantity of water in another beaker, which is at the temperature of the air—about 10° C.—and mix the two together, taking care to pour the hot water into the cold water, to avoid breakage. It will be found that the resulting temperature will be midway between 30° and 10°, or 20°, which shows that the heat which the hot water lost in falling from 30° to 20° is just equivalent to raise the colder water through the same number of degrees (10°). That is, the unit has the same value between 10° and 20° as between 20° and 30°.

It is well to vary this experiment in the following way:

EXPERIMENT 101.—Take a weight of water, W, at a temperature, T, and a different weight, W^1, at a lower temperature, t; then, if λ is the resulting

temperature of the mixture, we shall find that the heat lost by the hot water = heat gained by the cold water, $W(T - \lambda) = W^1(\lambda - t)$.

In both these experiments it should be observed that if the water is quite hot there will be considerable loss of heat while the hot and cold water are mixed and are being stirred, so that the temperature of the mixture shall be everywhere the same. This loss must be carefully estimated, by noticing the rate of cooling of the water.

It is well to repeat the preceding experiments, bearing in mind this correction for loss in cooling.

EXPERIMENT 102.—With a watch observe the time of mixture, the time of taking the temperature of the mixture, the time it takes the mixture to cool 1°, the time it takes to cool 2°, and so on. It will be found that the intervals of time between its falling through successive degrees—for instance, through three or four degrees—will be nearly uniform. If this interval is 10″ for the fall of one degree, and if it has taken 20″ to observe the temperature of the mixture, we know that the loss would be $\frac{20}{10} = 2°$, and the true temperature of the mixture would be $\lambda + 2°$, instead of λ which is observed.

Another source of error, which is common to all our experiments thus far in heat, is the loss of heat which comes from the absorption of heat by the containing vessels. For instance, when we pour the hot water into the cold water, a certain amount of the heat of the hot water goes into the vessel which contains the mixture. When this vessel is glass or thin metal, this loss is not great; if, however, the amount of water is great, and it is poured

into a thick copper vessel, this loss may be very large. In all cases, however, we must determine how much this loss of heat is.

EXPERIMENT 103.—Take some water at a temperature of $T°$ (about 50°) and pour it into the empty vessel used in the previous experiments, which is at the temperature of the air of the room in which we experiment. Find the resulting temperature and the rate of cooling, and correct as in the previous experiment for the rate of cooling. Call the weight of the water which is added, W. We then have: the heat lost to the vessel is $W(T - \lambda)$ units, and the heat gained by the vessel is $\lambda - t°$. Hence, to raise the vessel through 1° we need $\frac{W(T - \lambda,}{\lambda - t}$ units of heat. This number is called the specific capacity of the vessel. It is usual to divide this number by the weight of the vessel in grammes if it conducts heat uniformly throughout, as in the case of thin metal vessels. In this way we obtain what is called the specific heat of a substance, which is the heat required to raise one gramme of the substance through 1° C.

There is another source of error which arises from the use of the thermometer. This also absorbs a certain amount of heat while it is used in gently stirring the mixture and in observing its temperature. It also has a specific capacity which must be measured.

EXPERIMENT 104.—Pour a sufficient amount of water into the vessel, whose specific capacity we have found, to just cover the bulb of the thermometer. Call the weight of this water W grammes, and observe its temperature $t°$. Heat the bulb of

the thermometer to 100° C. in steam; dry it and hold it just above the water, and notice its temperature T. Then plunge it into the water of the vessel and notice the temperature. The thermometer in falling $(T - t)°$ raises the water and the vessel through $(\lambda - t)°$—that is, it gives up $(\lambda - t)$ units of heat to each gramme of water, and $(\lambda - t)$ $\times$ specific capacity of vessel to the vessel. Calling this specific capacity S, we perceive that the loss $W(\lambda - t) + (\lambda - t) S$, or $(\lambda - t)(W + S)$ units of heat, lowers the temperature of the thermometer $(T - \lambda)°$; hence the specific capacity of the thermometer, or the quantity of heat necessary to raise it 1°, is $\frac{(\lambda - t)(W + S)}{T - \lambda}$ units.

We are now in a condition to estimate the losses of heat in our first rough measurement of the mechanical equivalent of heat. Having carefully made all the corrections, upon a repetition of the experiment, we shall find that we approximate nearer to the true value, which, in English measure, we have said is as follows: 772 pounds raised one foot is the equivalent of the heat necessary to raise one pound of water 1° Fahr.; or, in French measure, 424 grammes raised one kilometre is the equivalent of the heat necessary to raise one gramme of water 1° C.

Our first experiment, however, is affected by the error in measuring the power consumed. Let us proceed to a more careful measurement, bearing in mind the fact that it is very difficult to measure heat on account of its rapid escape by radiation into the air, and by conduction into neighboring masses of matter.

The following experiment was performed by Hirn:*

EXPERIMENT 105.—Suspend an iron bar, *A* (Fig. 94), weighing 350 kilogrammes, by wires three me-

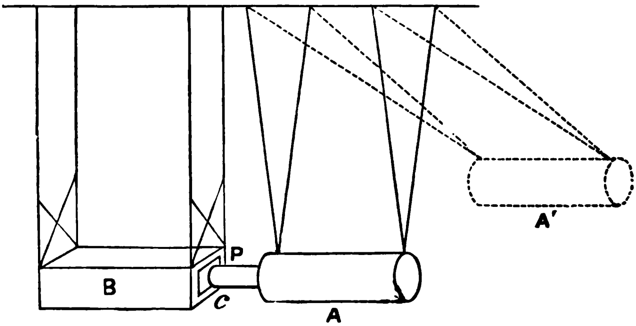

FIG. 94.

tres long; suspend also a block of stone, *B*, weighing 941 kilogrammes. (This is called the anvil.) This is provided with an iron plate, *C*. Between the hammer and the anvil a cylindrical piece of lead, *P*, weighing 2.948 kilogrammes, is held by a wooden fork. A cavity is made in the lead, as is shown in Fig. 95. The temperature of this cavity = 7.87° is obtained by means of the thermometer, and also the specific capacity of the lead.

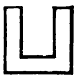

FIG. 95.

The hammer *A* is let fall from a height of 1.166 metres.

The height of the recoil after the blow is 0.087 metres.

The height of the recoil of anvil is 0.103 metres.

The work of the blow of the hammer is $L =$

* Müller's "Lehrbuch der Physik," ii, p. 896.

weight $\times$ height $= 350 \times 1.166 = 408.100$ kilogramme-metres.

The work done in the recoil of the hammer and the anvil is $l = 0.103\ (941 + 2.95) + 0.087 \times 350 = 127.677$ metre-kilogrammes.

The work used up in the compression of the lead is $L - l = 408.100 - 127.677 = 280.423$ metre-kilogrammes.

In the cavity of the compressed lead were placed 18.5 grammes of water at 0°.

Four minutes after the blow, the temperature was 12.10°.

Eight minutes after the blow, the temperature was 11.75°.

Therefore, during this time there was a cooling of 0.35°; and we have approximately, $11.75° : 0.35° = 12.10° : x$, or $x = 0.36°$, on the supposition that we have made in Experiment 102. Hence, the temperature of the lead at the moment of compression is $12.10° + 0.36° = 12.46°$.

The heating of the water by the blow $= 12.46° - 7.87° = 4.59°$. Hence, since the specific heat of lead $= 0.03145$ (see next experiment), $4.59 \times 2.948 \times 0.03145 + 12.46 \times 0.0185 = 0.659$ units of heat.

(The factor 0.0185 is the amount of water, 18.5 grammes, reduced to a kilogramme.)

We thus obtain $\dfrac{280.423}{0.656} = 427$ kilogramme-metres.

This experiment can be performed with good results by taking much smaller masses for both the hammer and the anvil. Suspend a small piece of lead before the anvil, by means of a thread, and let it drop quickly into a little calorimeter made of

sheet-brass, holding a small known quantity of water (see following experiment).

When a closer approximation is made in regard to the rate of cooling, 425.2 can be obtained.

Instead of taking the specific capacity of the lead vessel, we have used the specific heat of lead, which can be obtained from a table. It is always best to test the specific heat of the specimen of metal which is used. The following experiment is called the method of obtaining specific heat by mixtures:

EXPERIMENT 106.—Take a very thin copper or brass vessel, about one quarter the size of an ordinary tumbler, and find its specific capacity, which we will call S. Pour a known weight of water, W, into the calorimeter, and place a known weight, W^1, of the metal divided into small pieces in a thin glass test-tube; raise this metal to 100° C., by holding the test-tube in boiling water, then pour the metal quickly into the water which is in the calorimeter and at the temperature t of the room; then test the temperature with a thermometer whose specific capacity has been measured, and make a correction for cooling.

If we denote by x the specific heat of the metal, or the number of units of heat necessary to change the temperature of one gramme of the metal 1° C., we have heat lost by the metal = heat gained by the water, + the heat gained by the calorimeter + the heat gained by the thermometer, or,

$$W^1(100-\lambda)x = W(\lambda - t) + S(\lambda - t) + S_1(\lambda - t)$$
$$= (\lambda - t)(W + S + S_1).$$

Hence, $x = (\lambda - t)\dfrac{(W + S + S_1)}{W_1(100-\lambda)}$, in which S_1 is specific capacity of the thermometer, and λ is the tem-

perature of the mixture. Obtain in this way the specific heat of lead and of copper.

The results of Experiments 98 and 105 show us that there is an exact equivalence between mechanical motion and what we call heat. Wherever the energy resulting from the motion of bodies disappears, heat takes its place. Motion is therefore being continually changed into heat in all the operations of machines, and in the case of all falling bodies, and the heat is the equivalent of the work.

We have tested this great law, which is called the conservation of energy, as far as motion and heat are concerned. It is necessary to see how far it applies to other forms of energy, such as light, sound, electricity, and magnetism. Our experiments have also shown that perpetual motion is impossible, for we can not get any more work out of any combination of mechanical powers than we put into it. The term perpetual motion is often misunderstood. A water-wheel placed under Niagara Falls, if the materials would not wear out, might run forever. This, however, would not be perpetual motion, for the work done by the wheel is the equivalent of the work done by the water upon it. Neglecting the resistance of the air, a cannon-ball might be fired from a mountain at the equator with a sufficient velocity to make it revolve forever around the earth as a satellite. A velocity of five miles a second would accomplish this; for at the end of the first second

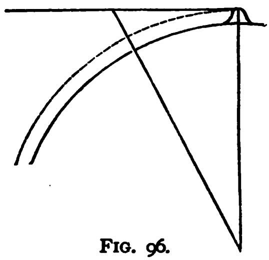

FIG. 96.

the ball would have fallen toward the center of the earth (Fig. 96) just as much as the earth curves away from a tangent line through the mountain, and there being no resistance, and consequently no heat as the result of motion, this motion would give up no part of itself, and the velocity of the ball would remain the same always. This is not an example of perpetual motion, for the motion having been once started, there is nothing to oppose it, a condition which is never met with in the case of any human mechanism.

We should obtain perpetual motion if we could arrange a series of inclined planes, so that a body after falling down one should rise along another to a greater height than that from which it fell, and so on. Taking the impossibility of perpetual motion as an axiom, we can prove that the velocity which a body acquires in falling down an inclined plane is equal to that which it would acquire if it fell down the height of the plane, for it would be just sufficient to carry the body up the height, neglecting friction.

If we could cut off the force of attraction between a pile-driver and the earth, we could lift it as we could a feather, without any effort, and letting it fall we could get any amount of work from it. A machine that could do this in an automatic way would be a perpetual-motion machine. The application of the principle of the conservation of energy and the impossibility of perpetual motion enable us to ascertain many laws of nature, and these laws are receiving every day fresh confirmation of their truth. We can prove, for instance, from the conservation of energy, that ice must melt under pressure, a fact which it would be difficult to test in a laboratory.

Water, it is well known, expands when it freezes. Suppose, therefore, we put some water in a cylinder, place a piston upon it, and heavily load this piston. Put a certain amount of freezing mixture around the cylinder. The water in freezing will lift this weight through a certain distance. Double the weight upon the piston: will the ice lift double the weight through the same distance as it did half of this weight? If it should do this, we should be getting out of the same freezing mixture an unlimited amount of work. Since we can not conclude that the amount of energy in a given amount of a freezing mixture is inexhaustible, we are forced to the conclusion that the freezing-point of water is lowered by pressure, or, in other words, that ice melts under pressure.

We have assumed in this example that the freezing mixture, which could consist of salt and pounded ice, can do a certain amount of work. Indeed, we know this from its effect in lifting a weight against the force of gravitation, by the expansion of ice which it forms. We commonly say that the freezing mixture abstracts heat from the water. In order to do work, there must be a difference of potential: we can say, therefore, that the heat potential of the water is higher than that of the salt and ice, and in the equalization of the difference of potential a certain amount of work is done. We are led to think of heat as a form of motion, since it is equivalent to motion and to mechanical work. Let us examine more carefully the amount of motion or heat, or the fall of heat potential when water congeals into ice.

EXPERIMENT 107.—Place in a calorimeter of known specific capacity, *S*, a known weight of water

W, at a known temperature T (about 50° C.); put into the water small pieces of ice which have been carefully dried with filter-paper; stir quickly until the ice is entirely melted, and observe the temperature of the water; then weigh the mixture; this will give the amount of ice, W_1, that has been added.

A certain amount of heat has evidently been used to change the ice at 0° into water at λ°.

Hence, heat lost by the water and calorimeter in falling from T° to λ° = heat required to convert W_1 grammes of ice into water at 0°, and afterward to raise W_1 grammes of water from 0° to λ°, or

$$(S + W)(T - \lambda) = W_1 L + W_1 \lambda. \quad \text{Hence,}$$

$$L = \frac{(S + W)(T - \lambda) - W_1 \lambda}{W_1}.$$

L is called the latent heat of water, and is the number of units of heat required to convert a unit weight of ice at zero into water at zero.

One must carefully make corrections for the cooling of the water if its temperature is higher than the air of the room before the ice is put in, and also for the heating of the mixture by the air if its resulting temperature is less than that of the room.

This latent heat is looked upon as the expression of a certain internal movement among the particles of the water.

The efficiency of a mixture of salt and ice in freezing neighboring bodies of water comes from its withdrawal of latent heat from the water in order that the liquefaction of the salt and ice can take place. Thus, when salt is placed upon sidewalks to melt the ice, it melts the ice in contact with it, but also freezes a certain amount of water in neighboring cracks and crevices.

We have spoken of the work that can be obtained from a freezing mixture; by freezing water and thus lifting weights, we cause in this way a difference of potential. By boring holes in granite and filling them with water, the fall of heat potential when the water freezes is sufficient to split the rock.

The great agent, however, which we use to work for us is steam. Let us examine the amount of work that is necessary to convert a unit weight of water at 100° C. into steam at 100° C.

Experiment 108.—Lead steam from a flask in which water is kept boiling, by a short tube, to prevent condensation, into another flask which contains a known weight, W, of water. To prevent the water from the condensing steam in the connecting tube entering the latter flask, provide the tube when it enters this flask with an arrangement similar to that in the figure, which consists merely of a larger piece of glass tubing which serves to catch the water coming from condensation at the end of the entering tube.

Fig. 97.

Let t be the temperature of the water and of the air, also λ the temperature of the mixture. After it has been corrected for the rate of cooling, weigh the water in the flask; the amount of steam that has entered can thus be found; call this W_1. The latent heat of steam will then be found by making the heat lost by the steam in condensing and in then falling from 100° to $\lambda° =$ the heat gained by the water in rising from $t°$ to $\lambda°$,

$$\text{or, } W_1 \left\{ L + (100 - \lambda) \right\} = W(\lambda - t),$$

$$\text{or, } L = \frac{W(\lambda - t) - W_1(100 - \lambda)}{W_1}.$$

Precaution.—The flask into which the steam is led should not be tightly corked. A way of escape should be allowed the excess of steam that is not condensed.

Since so much heat, or, in other terms, work is required to convert water into steam, we can conclude that more work can be got out of steam than out of boiling water. We are also led to suppose that there is greater movement of the particles of water in the shape of steam than when they are in the condition of particles in boiling water. Moreover, we must admit, when steam expands, and thus lifts a weight against the force of gravitation, for instance, that in doing this work it must part with a portion of its heat, otherwise we might obtain unlimited work from the same amount of heat. The amount of work we can get from steam at a certain pressure and temperature depends upon the fall of potential that we can obtain, just as in the case of a pile-driver, the amount of work we can get depends upon its height above the point where we wish to do the work; or, in the case of a waterfall, upon the vertical height through which the weight of water can descend. We could get all the work out of steam if we could deprive it of all its heat and utilize this heat completely.

The work that is done on a metallic bar, in communicating a certain amount of heat to it, can be made to have its equivalent in the work done in lifting a weight, *W*, against gravitation, and in doing

internal work among the molecules of the iron. It is evident, from the conservation of energy, that if we keep increasing the pressure against the end of a vertical rod, by increasing the weight upon it, the same amount of heat applied to the rod will not lift an indefinite weight. Otherwise, by merely expending a few foot-pounds of work in the shape of heat, we might get in return far more than we expended. We conclude, therefore, that a mass of iron inclosed under great pressure in a furnace will not expand so much by the application of a given quantity of heat as it would if the pressure were removed, and, consequently, would not melt at the same temperature as it would if the pressure were removed.

A mass of metal changes its dimensions with every change of temperature. By delicate means, even the influence of the approach of a person's body to within three feet of a metre-bar, a square centimetre in section, can be detected, and all very exact measurements of length must be conducted in a room of constant temperature. This molecular change, or evidence of work done among the particles of a body, evidently must greatly influence the glass and the mercury by which we make our heat-measurements.

EXPERIMENT 109.—To determine the co-efficient of expansion of glass of which a thermometer has been made, we proceed as follows: Blow a strong bulb about an inch in diameter on a strong piece of thermometer-tubing; cut the tube off four inches from the bulb; let the weight of the mercury in the bulb and tube be W, and observe the temperature of the air, t, which is the same as that of the mercury. Suspend the bulb in a large beaker, so

that the end or mouth of its tube projects over the side of the beaker. Then heat the water in the beaker to 75° C. or 80° C. (call this temperature $T°$) and catch the mercury that is expelled by expansion from the bulb. Weigh this amount of mercury, and call it w. If we now neglect the expansion of the glass vessel, we perceive that the weight $W-w$ of mercury that is left in the bulb at $t°$, becomes at $T°$ of the same volume as W. Therefore, $W-w$, on heating from t to $T°$, will increase by w. One volume will therefore increase $\frac{w}{W-w}$, and through one degree will expand by $\frac{1}{T°-t°}$ of $\frac{w}{W-w}$ or $\frac{1}{T°-t°} \cdot \frac{w}{W-w}$. This is the co-efficient of apparent expansion of mercury in a vessel made of this kind of glass. Let us call this co-efficient A. We shall find, from tables of expansion, that the co-efficient of real expansion of mercury is 0.0018 ($= B$). Hence we have $B - A =$ co-efficient of expansion of the glass. This instrument is called a weight-thermometer.

EXPERIMENT 110.—Determine the co-efficients of the absolute expansion of alcohol, between the temperature of the room and that of ice.

Weigh the weight-thermometer filled with the liquid at the temperatures 0° and $T°$. Call the weight of the liquid at $T° = W$, and the loss of weight at $T° = w$. We then have the apparent co-efficient of expansion $C = \frac{w}{WT°}$. We must add to this the co-efficient of the glass. A further study of the phenomena of heat leads us to the modern views of

molecular motion. In order to understand new theories, it is necessary to study first what has been well established. We shall, therefore, not dwell upon molecular theories, but proceed to show the relation between electrical work and heat and mechanical work, leaving to higher and more extended treatises the exposition of molecular theories.

CHAPTER XI.

ELECTRICAL WORK AND ELECTRICAL MEASUREMENTS.

WE began our study of motion with the law of attraction of gravitation, and we have found that the attraction between magnetic poles and between electrified bodies, or electrical currents, is expressed by a law similar to that of the attraction of gravitation.

In order to study the relation between mechanical work and electrical energy, we employ an instrument called a galvanometer. An instrument has been used in Experiment 71—not to measure an electrical current, but merely to indicate one. The instrument described in Experiment 71 is not suitable for the measurement of electrical currents; for it does not follow accurately the law of tangents. We have shown, in Chapter X, that the strength of a current passing through n turns of wire coiled upon a circular hoop is measured by the tangents of the deflection of a small needle placed at the center of the coil thus formed; or, $S = \frac{a\ T}{2\ \pi\ n} \tan \alpha$, in which a is the mean radius of the coil. T is the horizontal component of the earth's magnetism, n the number of turns, and $\pi = 3.1416$.

Such a tangent galvanometer can be very easily made by winding five to ten turns of No. 20 insulated wire upon a small hoop or wooden circle, such as the cover of a circular box, twelve to fifteen inches in diameter; fastening this hoop vertically to the end of a box; suspending a little magnet, provided with a plane mirror, by means of a silk fiber, at the center of the circle, and covering the circle with a plate of glass. A little lens, placed in front of the magnet, can be used to bring a spot of light to focus upon a piece of oiled paper or ground glass, according to the method described in Experiment 84. Rubber bands will serve to keep the plate of glass in position. The ends of the wire can be brought to the end of the box at *B B′*, Fig. 98. The box can be made steady by loading it inside.

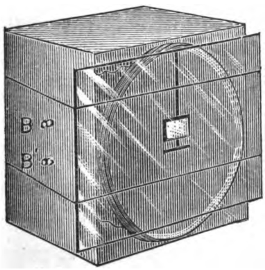

FIG. 98.

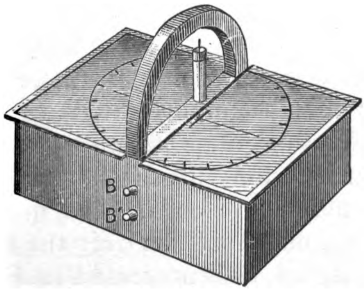

FIG. 99.

It is often desirable to be able to read the deflection of the magnet directly in degrees. For this purpose a cheap galvanometer can be constructed by placing the coil vertical in the middle of a box (Fig. 99); running a strip of board along its horizontal diameter, boring a hole in the middle of this

board, inserting a glass tube in this hole, suspending a magnet, with a pointer at right angles to it, by means of a silk fiber which runs through the glass tube, and is attached to a cork in the top of this tube. The pointer of the magnet moves over a graduated circle (of paper or of metal) which is mounted directly beneath the magnet. The box is then covered with two plates of glass, one on each side of the wooden strip which carries the suspension. The ends of the coil are brought to the sides of the box at B B^1.*

Tangent galvanometers are usually made with only comparatively few turns of wire in a coil of large radius. The magnetic needle is made very short in order that its movements may be confined to the small space at the center of the galvanometer-coil, where the field of force due to the coil is uniform. When a sensitive instrument is desired, which shall merely detect a feeble electric current, a galvanometer similar to that employed in Experiment 71 can be employed without the stationary magnets. Another little magnet should be hung upon the suspension, directly above the central magnet and outside the coils. The north pole of this outside magnet should be over the south pole of the lower magnet, as represented in Fig. 100, in which C represents an end-view of the coils. This arrangement is called an astatic system. The influence of the earth's magnetism is very feeble, since it acts in opposite directions upon the two magnets, and the lower magnet is ready to turn under the influence of feeble currents. The magnets should be rigidly connected

* The figure shows only the half of the coil which is outside the box.

together. Two sewing-needles, carefully magnetized and stuck through a piece of stiff straw, *a b*, will answer. A fiber of silk, without torsion, can then be attached at *a*. By suitably increasing the turns of wire upon the coils *C*, the instrument can be made very sensitive. We shall call it an astatic galvanometer, to distinguish it from a tangent galvanometer.

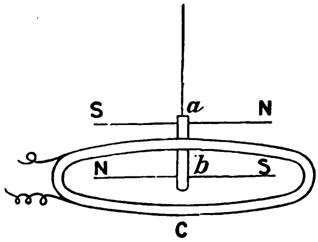

FIG. 100.

The metals which exhibit the peculiar property of magnetism are iron, steel, cobalt, and nickel. The earth itself can be likened to a great magnet. The position of the magnetic poles of the earth changes during a period of years. The angle which the compass-needle makes with the geographical meridian is called the angle of declination. This varies with the place and the time. Every piece of iron and steel is more or less magnetic. Any object made of the magnetic metals is magnetized to a certain degree by the earth's influence.

EXPERIMENT 111.—Examine the magnetic condition of a soft iron bar by presenting its ends to a galvanometer or compass-needle. Then hold it in the direction of the dipping-needle (see Experiment 83), strike it a sharp blow with a hammer, and examine its condition again. Repeat the operation, placing the bar in a horizontal position, in a line at right angles to the direction of the compass-needle. State the results of your investigation.

Direct experiment will show that there is no sub-

stance which will intercept or cut off the line of magnetic force. It is true that a piece of iron placed between a compass-needle and a magnet modifies the deflection of the needle, but this is due to the effect of the additional magnets formed by induction in the iron plate.

EXPERIMENT 112.—Study the effect of magnetic induction by placing a magnet at right angles to the magnetic meridian, and observing its effect upon a compass-needle when a sheet of soft iron is moved at regular distances between the pole of the magnet and the compass. Then place the compass inside a hollow cylinder, formed by bending the sheet-iron, and move the outside magnet. State the result of your investigation.

All bodies, and even gases, are more or less subject to magnetic influences, and are divided into two classes—paramagnetic, like iron and steel; and diamagnetic, like bismuth. Substances of the latter class tend to set themselves at right angles to the line joining two strong north and south poles of an electro-magnet. Diamagnetism is supposed to be due to a difference of effect upon the medium in which the substance is suspended, and upon the substance itself.

Ampère showed that the magnetism of a steel magnet could be explained by the action of electrical currents, which can be supposed to circulate around the axis of a magnet; for the action of a magnet can be imitated by a current passing through a wire coiled in the form of a spiral, thus constituting what is called a solenoid. This we have seen from Experiment 89. The direction of the current determines the nature of the magnetic poles—i. e.,

whether north or south. The poles of a solenoid, or of an electro-magnet, can always be determined by means of a small compass.

If a man were to place himself, in imagination, in the position of a very long, straight wire carrying an electric current, the current entering by his head and departing by his feet, a magnet suspended freely before him would turn, under the action of the current, the pole which points north toward his right hand.

We have shown that an electro-magnet resembles a steel magnet. It has two poles, which depend upon the direction of the current through the coil. Two parallel coils, therefore, in which the current passes in the same direction should attract each other, for the north pole of one will attract the south pole of the other. Two parallel coils in which the current passes in opposite directions will repel each other, for poles of the same name will be brought opposite each other. We conclude, in general, that parallel currents in the same direction attract each other, and parallel currents in opposite directions repel each other.

We have, therefore, a general relation between electricity and magnetism. We can produce magnetism by an electrical current. We shall show subsequently that we can produce electricity by the movement of a magnet, and we shall find an equivalence between the electricity produced and the movement which produces it, thus finally bringing our study of electricity and magnetism back again to the study of motion in general.

In the first place, it may be stated as a general law that any change in the relative position of the

particles of two different substances is accompanied by a manifestation of electricity. We can think, to fix our minds, upon a certain modification of the forces of attraction between the particles of two different substances which is always accompanied not only by heat, but also by a difference of electrical potential. Thus, friction produces this difference of potential; and the difference of action of a liquid upon two metals dipped in the same liquid modifies the attraction between the particles of the metal, and we have also a modification of the forces of attraction between the molecules of the liquid and the molecules of the metals. We have treated electrical force and magnetic force, together with gravitation force, simply as what are called forces of attraction, the general law of attraction being of the same form; the force in each case varying inversely as the square of the distance between the attracting configurations. In proceeding to study magnetic and electrical phenomena, we shall see that we are still studying potential and kinetic energy, due to the force of attraction. Although we do not know what magnetism is and what electricity is, we must also reflect that we do not know what gravitation is. We can, however, study the changes or transformations of energy which take place, and the relations between these various manifestations of attraction.

We have already seen that a current of electricity can be considered as the evidence of the fall from one electric level to another, and is the evidence of work being performed. A difference of electrical level can always be produced by means of two different metals, or, in general, by means of

any two substances brought in contact with each other, either directly, or by means of a liquid or liquids. Whenever the forces of attraction between the particles of two dissimilar bodies are modified in any way, a difference of electrical level is produced.

EXPERIMENT 113.—Place a strip of sheet-zinc, about eight by four centimetres, in acidulated water formed by adding one part of sulphuric acid to eight of water. Bubbles of hydrogen gas will collect on the zinc, and the zinc will be gradually consumed.

Place a copper strip, of the same size as the zinc strip, also in the liquid, but not touching the zinc, and note what happens.

Now connect the zinc and copper strips outside the liquid by bending them until they touch. Observe what happens. Weigh the zinc and copper electrodes or plates after an interval of an hour.

Amalgamate a zinc plate by carefully cleaning it first in acidulated water, and rubbing mercury upon it with an old tooth-brush. Replace it in the water, opposite the copper, and observe what happens.

Weigh the amalgamated zinc before it is put in the acidulated water in company with the copper, and after an hour has elapsed. (In weighing, be careful that the amalgamated zinc does not come in contact with the metal of the balance-pans; place a piece of paper beneath it.)

EXPERIMENT 114.—Connect the zinc with the copper by means of the terminals of a galvanometer. The electrical current will be shown by a deflection of the magnetic needle.

The number of different forms of galvanic cells is very great. The following are those which are generally used in laboratories:

EXPERIMENT 115.—Make a gravity-cell in the following manner: Coil one end of a bare copper wire into a flat spiral (Fig. 101). Make the plane of this spiral perpendicular to the straight portion of the wire. Cover this straight portion with some liquid-proof insulator, like gutta-percha and tar, except at its point of connection with the spiral. The spiral, which is merely to obtain considerable surface of copper, is then laid at the bottom of a glass vessel, which can be made by cutting off the top of an ordinary bottle. The spiral is then covered with crystals of sulphate of copper to the height of two or three inches, and the vessel is filled with water. In the water, about four inches from the layer of copper, is hung a piece of zinc with a suitable connection for attaching a wire to it. After a while the sulphate of copper dissolves, and sulphate of zinc is also formed. The difference in specific gravity of the two liquids keeps them separated. Instead of the zinc, one can use a piece of iron. In this case it is best to partially fill the jar with sulphuric acid and water—one part of acid to twelve of water—and afterward drop into this crystals of sulphate of copper.

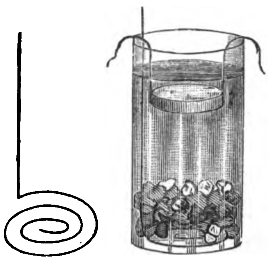

FIG. 101.

The Daniell cell, which is most commonly used for delicate electrical measurements, differs from

the gravity-battery in having the piece of zinc and the surrounding sulphate of zinc separated from the sulphate of copper by a porous jar. The zinc is carefully amalgamated by cleaning it with dilute sulphuric acid and immersing it in mercury, to prevent unequal action of the sulphuric acid upon the zinc.

The Bunsen cell consists of an amalgamated cylinder of zinc immersed in sulphuric acid and water (one part of acid to twelve or fourteen of water), and a stick of carbon immersed in concentrated nitric acid contained in a porous jar, the porous jar being placed inside the zinc cylinder. This cell is very objectionable, on account of the fumes of nitrous acid which arise when the cell is working. These fumes quickly injure all apparatus made of brass, and also tools. It is usual, therefore, to replace the nitric acid by a saturated solution of bichromate of potash and a certain amount of sulphuric acid. The ordinary proportion is: two ounces of bichromate of potash dissolved in a quart of water, to which is added two fluid ounces of strong sulphuric acid.

The Leclanché cell is in common use for ringing bells and for the telephone service. It consists of a cylinder of zinc immersed in a solution of sal-ammoniac, and a bar of carbon in a porous jar, surrounded by black oxide of manganese and powdered carbon. It is a very inconstant cell, and its strength decreases rapidly when the poles are connected by short, thick wire. When it is used, the circuit should be closed only for a moment.

EXPERIMENT 116.—Make a water-battery. This consists of zinc and copper elements immersed in

pure or distilled water. Obtain a gross of small bottles of a druggist. Two and a half inches in height, with a mouth one inch in diameter, is a convenient size. Dip the mouths of the bottles in melted paraffine to a distance of one inch, and place them in a box provided with a cover, to keep out the dust. Solder a strip of zinc four inches broad to a strip of copper of the same breadth. Cut the united strips into small couples like *C Z*, Fig. 102. Bend these couples and place the zinc end in one

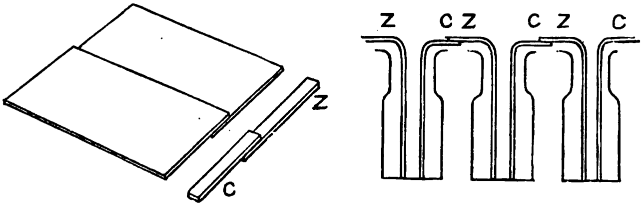

FIG. 102.

bottle and the copper in the adjoining one, and so on. A large number of these cells can be quickly and cheaply made, and a great difference of electrical potential can thus be obtained. From eighty to one hundred of these small cells will serve to charge Coulomb's balance (Experiment 88), and also the electrometer (Experiment 94). To employ the battery for this purpose, connect the last copper strip with the inside of the Leyden-jar, and the last zinc strip with the outside of this jar. This use of a water-battery shows that the poles of a battery have opposite electrical charges—one positive and the other negative.

We have seen that we can measure the strength of an electrical current by the expression $S =$

$\frac{Ta}{2 \pi n} \tan \alpha$. The question arises, How can we measure the work which a current of strength S does? It is not difficult to show that a wire becomes heated through which a current of electricity passes.

EXPERIMENT 117.—Connect four Bunsen cells in series—that is, the carbon of one cell with the zinc of the next, and so on (Fig. 103). Make the conductor which connects the last zinc and the last carbon of thick copper wire, except at one place, where the ends of the thick copper wire are connected with a very fine platinum wire about ten centimetres in length. On making the circuit, the platinum wire will glow. Place it in a small copper calorimeter similar to those which were used in the determination of specific heat, taking care that the platinum wire does not touch the sides of the calorimeter. Fill the vessel with alcohol and place a thermometer in it. Place another thermometer in the liquid of the battery, next the carbon element. In a short time both of these thermometers will indicate heat. It will be found that whenever, by changing the length of the platinum wire, the heat is decreased in the cell, it is increased between the poles of the battery—or, in other words, in the calorimeter. The short, thick copper conductors are not sensibly heated.

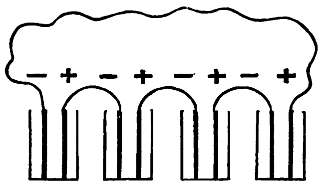

FIG. 103.

This heat is an evidence of work, and has its equivalent in work, which in the case of a galvanic battery is due to the difference of potential created

in some unknown way between two different metals. The amount of heat developed in the circuit outside the battery depends upon what is called the resistance of the circuit. This consists of the extent of liquid through which the electrical action takes place in the cells, and also of the length and section of the interpolar wire. In order to obtain the work of an electrical current we must first obtain an idea of electrical resistance.

EXPERIMENT 118.—Arrange a galvanic cell so that the resistance of the cell can be neglected. In a vessel containing bichromate of potash immerse a plate of zinc and a plate of carbon at a certain distance from each other, and connect them by a certain interpolar wire of which the wire of a galvanometer forms a part. Place the galvanometer so that the plane of the coil is vertical and the magnetic needle is in the plane of the coil. The strength of the current will be expressed by $S = \frac{T a}{2 \pi n} \tan \alpha$.

When the interpolar wire is long, it will be found that the distance of the plates in the liquid can be doubled without affecting the strength of the current, as is indicated by the galvanometer. We therefore conclude that this liquid resistance can be made inappreciable in our experiment by experimenting with considerable interpolar resistance, in comparison with which the battery or liquid resistance can be neglected.

EXPERIMENT 119.—Therefore, taking suitable lengths of copper wire, interpose them in a circuit consisting of a battery and a galvanometer, and observe the relation between the strength of the currents and the length of the wires. It will be

found that the tangents of the angles of deflection are inversely as the length of the wires of the same section. This shows that—

$$\frac{S}{S^1} = \frac{\frac{T a}{2 \pi n} \tan a}{\frac{T a}{2 \pi n} \tan a^1} = \frac{\frac{E}{R}}{\frac{E}{R_1}} = \frac{R_1}{R},$$

for the factor $\frac{T a}{2 \pi n}$ is constant for the same galvanometer, and for the same magnetic field of the earth. It will, therefore, cancel out when we take the ratio of the strength of the currents. We know also that the expression for the strength of the current must be a constant quantity E divided by the resistance, for the ratio between the currents is that of the inverse ratio of the length of the wire. The constant factor $\frac{T a}{2 \pi n}$ is called the reduction factor of the galvanometer, because it enables us to reduce our expression for the strength of an electrical current to absolute units—that is, to distances in centimetres, and to weights in grammes. An observer at any point on the earth's surface, knowing the reduction factor of his galvanometer, can compare his results with those of an observer situated at some other point of the earth's surface. The constant factor E evidently depends upon the difference of potential of the particular cell that is employed. The strength of a current, therefore, we can conclude is expressed as $S = \frac{E}{R}$, in which the resistance of the battery is neglected in comparison with the interpolar resistance, and the resistance of the

interpolar wire depends merely upon its length, for we have supposed that its section is uniform.

EXPERIMENT 120.—Repeat the previous experiment, keeping the length of a wire constant, but using wires of different cross-section. We shall find that the greater the section the greater the current, or $\frac{S}{S_1} = \frac{\tan a}{\tan a_1} = \frac{r}{r_1}$, in which r and r_1 are the radii of the cross-sections.

Hence, we conclude that the strength of the currents varies inversely as the resistances interposed in the circuit, and that these resistances vary directly as the length of the wire, and inversely as their section; or, the resistance $R = C\frac{l}{s_1}$, C being a constant factor depending upon the kind of wire that is used. In obtaining the ratio between the resistances of any lengths of the same kind of wire, it is evident that this factor will cancel out. If wires of different metals are employed, this factor will be different for the different metals. It is called the specific resistance of a metal. In order to obtain a standard by means of which we can estimate specific resistance, we must determine what our unit of resistance shall be. It will be remembered that, in order to determine specific gravity, we had to know the weight of a cubic centimetre of water at 0° C., and in order to determine the specific heat of a metal we were obliged to know the amount of heat necessary to raise a gramme of water 1° C. In the same way we must base specific resistance upon some unit which will be universally recognized. The simplest unit of resistance is that proposed by Siemens. It consists of a metre-length of mercury

contained in a tube one square millimetre in section. This is called Siemens's mercury-unit. Its value is $\frac{95}{100}$ of the B. A. unit, or British Association unit, which is the one usually adopted.* The explanation of how the B. A. unit is measured is somewhat difficult, and we shall defer it until we have seen how resistances in general can be measured; and how resistance finally can be expressed in absolute measure—that is, in units of space, time, and mass. Our final object is to connect the phenomena of electricity and magnetism with those of motion and of heat, and to illustrate the conservation of energy.

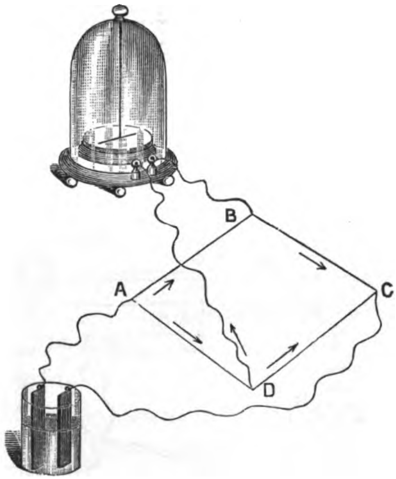

Fig. 104.

EXPERIMENT 121.—Experiment 117 shows that the amount of heat developed in an electrical circuit depends upon the length of wire or resistance. In order, therefore, to determine the amount of heat, and in this way measure the mechanical equivalent of it, we must be able to determine the resistance of the circuit. The simplest method is by what is called

* At the Electrical Conference held in Paris, April, 1884, a column of mercury 106 centimetres in length and one square millimetre in section at 0° C. was adopted as the ohm.

the Wheatstone bridge. Arrange a parallelogram, $ABCD$ (Fig. 104), of uniform copper wire, in the manner shown in the figure, leading the poles of the battery to the points A and C, and attaching the wires of the galvanometer at B and D. It will be found by experiment that the length AB will be to the length AD as BC is to DC, or $\frac{AB}{AD} = \frac{BC}{DC}$, or $AB = \frac{BC}{DC} AD$. If, therefore, we knew the electrical resistance of AD, we could obtain that of AB, for the ratio of the resistance of BC to DC is evidently the same as that of their length, since the section of the copper wire is everywhere the same; and these can be readily measured. The resistance of AD can evidently be a Siemens mercury-unit, and we can obtain AB in terms of it.

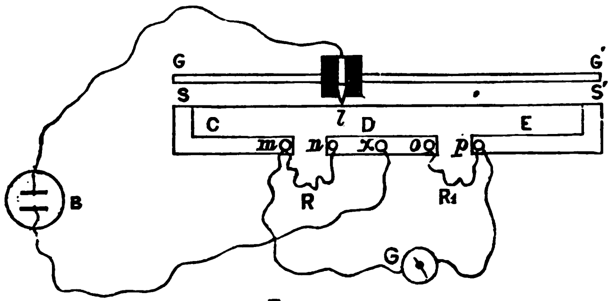

FIG. 105.

EXPERIMENT 122.—A simple Wheatstone bridge can be constructed as follows: Screw three flat pieces of brass or copper plate, C, D, and E, to a board, as in Fig. 105. Solder a German-silver wire, SS^1, to C and E. Provide binding-screws at m, n,

x, o, and p. One pole of the battery is attached permanently at the point x, while the other, by means of a block of wood sliding on a guide, $G\,G$, can be made to touch $S\,S^1$ at any point. When no current flows through the galvanometer G, we have $\frac{S\,l}{R} = \frac{l\,S^1}{R_1}$, or $\frac{S\,l}{l S^1} = \frac{R}{R_1}$. Knowing the ratio of $S\,l$ to $l\,S^1$, and knowing also the value of R_1, we can obtain R.

A table of the electrical resistance of copper wire of various diameters is given in the Appendix. From this table a set of resistance-coils can be constructed which will be accurate enough for ordinary experiments. When great accuracy is required, these resistances must be compared with some standard B. A. coils. Other resistances, based upon these calculated ones, can then be made by the Wheatstone bridge.

EXPERIMENT 123.—Make a set of resistances. Solder a piece of copper wire of large diameter to one end of a fine copper wire. Measure a metre of the wire and interpose it as a resistance to be measered in the Wheatstone bridge. Having measured the required resistance, solder a large copper wire to the end of the wire which you cut, so that both ends of the resistance may now be provided with these large wires, which are useful for connecting with the binding-cups. Wind these resistances upon spools, and slip pieces of rubber tubing over the spools, in order to confine the wire. It is useful to obtain an idea of the average resistance of the wire you are employing to make these resistances, so that you may not cut the wires unnecessarily short or long.

EXPERIMENT 124.—Instead of placing the galvanometer between B and D, as is usual in the Wheatstone bridge, place it between A and D (Fig. 106) as a resistance to be determined, and put a key in its place between B and D. Obtain the ratio $\frac{AD}{AB} = \frac{DC}{BC}$. When this ratio is true, the deflection of the galvanometer should not change when the circuit BD is made or broken. Therefore the resistance of the galvanometer, or $AD = \frac{DC}{BC} AB$.

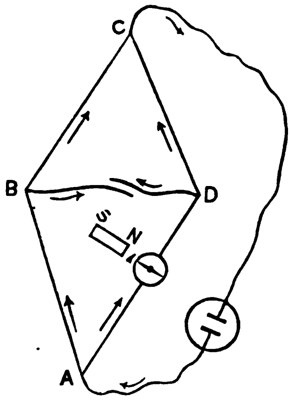

FIG. 106.

It is generally necessary to place a magnet outside the galvanometer to bring the deflection of the needle of the latter within range (if the galvanometer is a reflecting instrument).

The electrical resistance, R, of a cylindrical conductor is directly proportional to its length, l, and inversely to its sectional area, q; or, $R = k\frac{l}{q}$. The factor k is called the specific resistance of the conductor. The specific resistances of various bodies can be conveniently referred to that of Siemens's mercury-unit, which consists of a metre-length of mercury, one square millimetre in section, at 0° C. The specific resistance of this unit at this temperature is taken as 1. Table VI, Appendix, gives the galvanic resistances of the common met-

als compared with that of a column of mercury at 0° C.

EXPERIMENT 125.—Determine the specific resistance of a length of copper wire at the temperature of boiling water.

An increment of temperature of 1° C., at a mean temperature, produces an increased resistance in pure, solid metals of about 0.4 per cent—German-silver about 0.04 per cent, and mercury about 0.08 per cent.

Example.—The resistance of five metres of pure copper wire 0.4 of a square millimetre in section, at $0° = \frac{5 \times 0.0162}{0.4} = 0.2$ Siemens, and at 10° C., $0.2 + 0.2 \times 0.004 \times 10 = 0.208$ Siemens.

The length of copper wire should be attached to large connecting pieces of copper, the resistance of which can be neglected, and immersed in a beaker of water which had first been raised to the boiling-point. Ascertain how nearly the actual resistance of a piece of copper wire agrees at the temperature of 100° C. with the calculated value.

The arrangement called Wheatstone's bridge, after its inventor, Sir Charles Wheatstone, can be

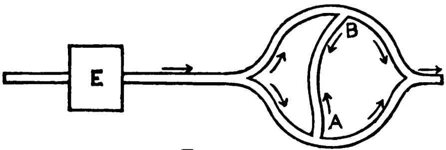

FIG. 107.

explained by the theory of the fall of potential. Let us recur to the analogy of the engine pumping water, and suppose that a pipe should connect some

point, *A* (Fig. 107), on the main through which the water is rushing from the engine *E*, with some point *B*, another branch of the main through which the water also issues. If the pressure at *A* is the same as the pressure at *B*, or, in other words, if there is no difference of level between *A* and *B*, or no difference of potential, there will be no flow from *A* to *B*, or from *B* to *A*. A small float placed in the water which fills the pipe *A B*, would be stationary wher-

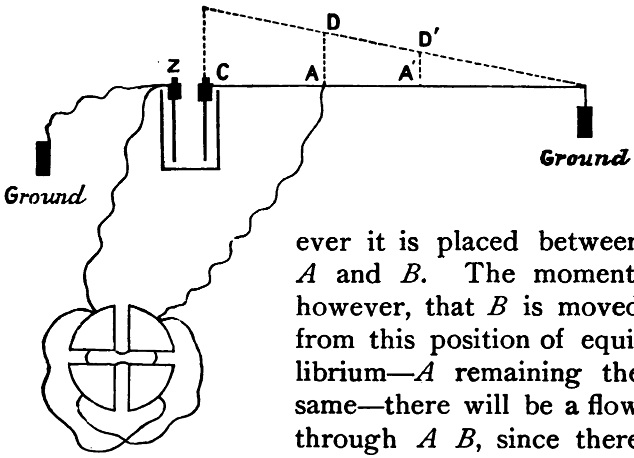

FIG. 108.

ever it is placed between *A* and *B*. The moment, however, that *B* is moved from this position of equilibrium—*A* remaining the same—there will be a flow through *A B*, since there is now a difference of level. We must not infer from this analogy that there is a flow of electricity like that of water. This analogy, however, leads us to try the following experiment:

EXPERIMENT 126.—We have seen that there is a difference of potential between the two poles of a battery. We should therefore expect to find a difference of electrical level between the zinc pole *Z* (Fig. 108), which is connected with the ground, and

any point A along the conductor which connects the positive pole C with the ground. With the electrometer which is used in Experiment 94, test the difference of potential between the zinc pole Z and any point, such as A. To do this, connect one set of the quadrants with Z and the other with the point A, and notice the swing of the needle. Lay off perpendicularly to A with any convenient unit, the distance $A\ D$ equal to the deflection of the electrometer, laying off also the distances $A\ C$, $A'\ C$, equal to the lengths of wire or resistances between the pole C and the points A, A', etc. On connecting the points D, D', etc., it will be found that they will lie on a gradient which rises from the point where the line is grounded at E to the positive pole of the battery. It is evident, therefore, that this method would enable us to discover a complete break in a cable; for, having obtained several points on the gradient, such as D and D', on each side of certain resistances interposed between the end of the cable and the positive pole of the battery, we can lay off these resistances and deflections on a suitable scale on paper, and extend the line of gradient until it strikes the point on the extended line of resistance, or cable, where the contact with the water, or, in other words, the earth, occurs. This experiment requires five or six cells.

The fall of electrical potential can also be shown by means of an electrical machine. Descriptions of the many forms of electrical machines can be found in any treatise on natural philosophy. For our present purpose we merely wish to show the close analogy between the kind of electricity produced by the motion of the hand in turning such a

machine, and the electricity produced by the mere contact of dissimilar metals in a battery.

Connect the terminals of the machine (Fig. 109) with the ground. Interpose certain resistances at *A*, and charge a Leyden-jar, similar to that which

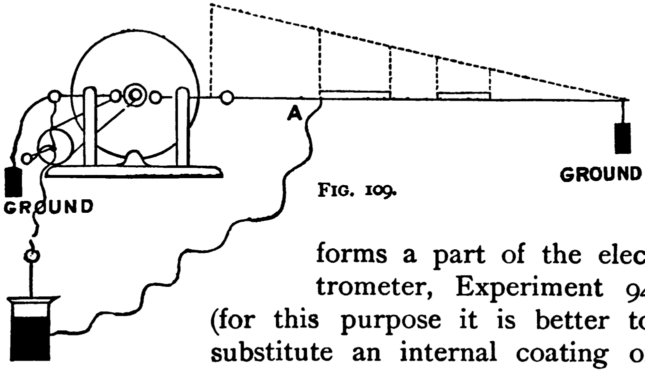

FIG. 109.

forms a part of the electrometer, Experiment 94 (for this purpose it is better to substitute an internal coating of tin-foil for the sulphuric acid). To charge the jar, connect the internal coating with one pole—the negative—of the machine, and the other with a point such as *A*. If we should now connect the charged coatings of this Leyden-jar with our electrometer terminals, the needle of the instrument would receive too great an impulse. We shall, therefore, have to connect one set of the quadrants with a metallic plate, which is mounted on a glass insulating stand, readily made from a piece of glass tubing and a wooden base. Another exactly similar insulated metallic plate is placed directly opposite the first, at a suitable distance, this distance being regulated by the effect upon the electrometer. In this way the charge of the metallic plate connected with the Leyden-jar induces a charge upon the opposed metallic plate, which is proportional to the charge

of the Leyden-jar, and also to the difference of potential between one pole of the electrical machine and the point touched upon the line running from the other pole to the earth. Laying off the distances and deflections, as in the experiment with the battery, we find the same law of fall. On connecting the terminals of a Leyden-jar with each other, we perceive that a spark passes. We know in this case that a difference of potential exists in the charged jar, and we perceive, by the light and heat (for this spark can ignite gunpowder), that work is done by the sudden fall of this potential. If we had the means of measuring the heat of the spark, we could see whether the mechanical work done in producing the electrical difference of potential is equivalent to the heat produced. Let us now measure the heating effect of an electrical current produced by an ordinary battery. We have shown that this heating effect depends upon the resistance of the circuit. It is proved by direct experiment that the heating effect, H, is equal to the square of the current, S, multiplied by the resistance, R, of the circuit, and multiplied by the time, t, or $H = S^2 R t$.

EXPERIMENT 127.—To prove this, connect four Bunsen cells, or their equivalent, by thick copper strips, interposing a galvanometer, and certain lengths—from three to ten centimetres—of fine platinum wire, of which the resistances have been measured. These lengths of platinum wire are immersed in a calorimeter filled with distilled water or alcohol (if alcohol is used its specific heat must be known, or the amount of heat absorbed by the alcohol compared with that of an equal

weight of water). If we call the weight of the water heated W, and h its temperature, we shall find $\frac{J\,W\,h}{J\,W\,h}=\frac{m^2\tan^2 a}{m^2\tan^2 a_1}\;\frac{R\,t}{R_1\,t}=\frac{S^2\,R\,t}{S_1^2\,R_1\,t,}$ in which we have called the mechanical equivalent of heat J; m, the reduction-factor of the galvanometer employed (whose coil should be one or two turns of copper wire about twelve centimetres radius), and a and a_1, the different deflections with different lengths, R and R_1, of wire heated, t and t_1 being the times during which the current runs. In proving this, it is evident that we do not need to know the mechanical equivalent of heat, J, or the units in which we estimate the strength, S, of the current, or the unit of resistance, since the numerical values of these units would cancel out in the above ratio. If we knew the value of the unit of resistance expressed in terms of the units by which we express motion, we could obtain the value of the current in the same absolute currents, for $S^2=\frac{J\,W\,h}{R\,t}$, in which everything is expressed in absolute units. Moreover, since $S^2=\frac{E^2}{R^2}$ (Ohm's law), we have $\frac{E^2}{R^2}=\frac{J\,W\,h}{R\,t}$, or $E^2=\frac{J\,W\,h\,R}{t}$. From this we perceive that we can also obtain the electro-motive force E in absolute units if we know the unit of resistance. Conversely, if we could measure the unit of resistance exactly, we could obtain the mechanical equivalent of heat, J. To express, therefore, our electrical measurements in the same units that we have used in the expression of mechanical work and its equivalent heat, we must know the value of the

electrical unit of resistance. To obtain an idea of the method employed, let us first see what effect will be produced by moving a conductor, of which the ends are connected, in a magnetic field. We shall take, in the first place, the magnetic field of the earth.

EXPERIMENT 128.—Wind around an ordinary wooden hoop a number of turns of moderately fine insulated copper wire, tying the wire at certain intervals to the hoop by strings. Leave the ends of this wire long, and connect them with an astatic galvanometer of low resistance. Lay the hoop horizontally upon a table and turn it quickly completely over, so that it may rest upon what was before its upper side. The needle of the galvanometer will be deflected, thus showing that an electrical current is produced by the motion of the conductor in the earth's magnetic field. Notice how much this deflection is, then place the hoop vertical, and perpendicular to the direction of the magnetic needle. Turn the coil quickly around a vertical axis, through 180°, and note the deflection. By the first operation we have cut the vertical lines of the earth's magnetism twice (each half-circumference cutting them all), and by the second we have cut the horizontal lines twice. We shall find that if we divide the deflection produced by cutting the vertical lines of force by a given coil by that obtained from cutting the horizontal lines of force by the same coil, we shall obtain the tangent of the angle of dip, or the inclination of the total magnetic force to the horizon. We can conclude from this that the strength of a magnetic field can be measured by cutting its lines of force by a coil of a known number

of coils, and a known radius. That different coils produce different effects, when revolved in the above manner in the earth-fields, can be readily learned from experiment. There must, therefore, be a constant, which belongs to each coil. To obtain the strength of any magnetic field, proceed as follows:

EXPERIMENT 129.—Make a small bobbin of fine insulated copper wire (twenty turns on a circle of wood about one centimetre in diameter). Connect the terminals of this coil with an astatic galvanometer of small resistance, and place the coil parallel to the plane of the pole of an electro-magnet, the area of which is considerably larger than that of the circle of the small bobbin. Quickly reverse (see Appendix) the current in the electro-magnet, and notice the swing of the galvanometer-needle. Afterward, suddenly revolve the bobbin through 180° around an axis perpendicular to the axis of the electro-magnet, while the current is constantly flowing through the electro-magnet. It will be noticed that the induction-currents are the same in both cases. Hence, the effect produced by reversing the current is the same as cutting the lines of magnetic force twice by the bobbin. This shows us how we can refer our measures of the strength of a magnetic field to the strength of the earth's field, and therefore to absolute measure. Inclose in the circuit of the small bobbin a large hoop covered with twenty turns of insulated wire. Mount this hoop vertically, with its plane east and west. Turn it quickly through 180°. The induction-current thus produced will be due to cutting the horizontal lines of force of the earth's magnetism

twice; for each semi-circumference cuts them through 180°.

The operation of turning a coil of a certain radius and a certain number of turns of wire in the earth's magnetic field, we perceive, will give us an electrical current through a certain resistance. Since we know from Ohm's law that the current is equal to the electro-motive force divided by the resistance of the circuit, the resistance of the circuit not being altered when we turn a coil in a magnetic field, the electro-motive force must be proportional to the strength of the field, and to the number of lines of force cut by the revolving coil. If the area inclosed by one turn of wire of a coil is πR^2, then $2\pi R^2 \times H$, where H is the strength of the field, represents the number of lines of force cut by turning this coil through 180°. If there are n turns of wire in the coil, the number of lines of force cut will be $2\pi R^2 \times n \times H$.

Hence, if we should inclose two coils in the same circuit with a galvanometer, and, placing the plane of the larger of the two horizontal, should quickly turn this through 180°, and, having placed the smaller with its plane perpendicular to the lines of force in another uniform magnetic field like that between two magnetic poles, the lines of force being straight, should also turn this through 180°, we should have—

$$\frac{2\pi R^2 \times n \times H}{2\pi R_1^2 \times n^1 \times H_1} = \frac{a}{\beta} =^*$$

$$\frac{\text{number of lines of force of earth inclosed}}{\text{number of lines of force of magnetic field inclosed.}}$$

* In this equation a and β are the sines of one half the angle of deflection of the galvanometer, which for small deflections can be

Knowing the strength of the vertical component H of the earth's field, we can readily obtain the strength H_1 of any other field.

The method of determining the unit of electrical resistance which is called the ohm depends upon a process of cutting the lines of force of the earth. A circular coil of radius K, and including a length L of wire, is rotated around a vertical axis which is perpendicular to the horizontal lines of force of the earth's magnetism.

In one complete revolution each half of the coil will cut the horizontal lines of force twice. We can satisfy ourselves by experiment that a certain electro-motive force is generated by cutting the lines of magnetic force, for we perceive that we obtain an electrical current by this means through a certain resistance. Ohm's law, or $S = \frac{E}{R}$, applies to this current. The factor R is fixed by the length of wire we use in our experiment. The factor E, or the electro-motive force, can only depend upon the strength of the magnetic force, or, in other words, the number of lines of magnetic force which are cut by the revolution of the coil, and the number of revolutions. Calling the radius of our coil K, it is evident that the number of lines of force which pass through one coil of this radius would be $\pi K^2 H$, where H is the strength of the earth's field or the number of lines of force to the unit area. If this coil revolved once, each half of the

taken as proportional to the swings themselves. If the small coil is quickly removed from the magnetic field parallel to itself, the expression becomes $\frac{2\pi R^2 \times n \times H}{\pi R_1^2 \times n^1 \times H_1} = \frac{\alpha}{\beta}$.

coil would cut this number of lines of force twice, and therefore the whole coil would cut them four times. If n were the number of turns of the coil in a second, we should, therefore, cut $4 \times \pi K^2 \times H \times n$ lines of force. If now we have N coils instead of one coil, and if K is the mean radius of all the coils, each coil will evidently cut the lines of force four times, and, therefore, the whole number will be $4 \pi K^2 H n N$. This is the electro-motive force which produces the current S through the resistance R, which includes the resistance of the galvanometer and the revolving coil and connecting wires. The current will then be $S = \frac{4 \pi K^2 n H N}{R}$. This current, on the other hand, will produce a magnetic field at the center of the revolving coil. It will be noticed, in turning the coil of the preceding experiment from east to west, that the deflection of the galvanometer is in one direction, and in the opposite when the revolution is completed from west to east. The current, therefore, reverses its direction with each half-revolution of the coil. If a compass were placed at the center of a stationary vertical coil, and an electrical current were quickly sent through it, first in one direction and then in another, we find that the needle would remain at rest, for the north pole of the magnet is acted upon alternately by a north and south pole of the electro-magnetic coil constituting the galvanometer, and, being thus solicited by opposite forces, would remain at rest. Suppose, however, we should hang at the center of the revolving coil, by a fine silk fiber which a large number of revolutions would not twist sensibly, a small mag-

net provided with a mirror. If, now, each time the current should reverse, we should accompany this reversal by a corresponding turn of the coil, it is evident that the little magnet would be deflected from the magnetic meridian, and the action of the forces upon this little magnet would be the same as in the ordinary galvanometer, or $S \frac{2\pi K N}{K^2} \mu l \cos\phi = H \mu l \sin\phi$; or, substituting the value of $S = \frac{4\pi K^2 n H N}{R}$, we have $\frac{4\pi K^2 n H N}{R} \times \frac{2\pi K N \mu}{K^2} l \cos\phi = H \mu l \sin\phi$ (calling H the intensity of the earth's magnetism, μ the magnetism of the magnet, l the half-length of the magnet). In this equation R is the only quantity that is unknown. On solving, we find $\frac{8\pi^2 K n N^2}{R} = \tan\phi$, or $R = \frac{8\pi^2 K n N^2}{\tan\phi}$. The whole length of wire, $L = 2\pi K N$ (that is, N times the average circumference of each coil). Hence, we can express R in terms of this length, or $R = \frac{2 L^2 n}{K \tan\phi}$. It will be seen from this expression that the electrical unit of resistance is expressed as a velocity, for n is the velocity of revolution, and $\frac{2 L^2}{K \tan\phi}$ is a co-efficient which can be made unity by properly taking the length of wire and the radius of the coil.

We see, therefore, that from the motion of a wire in a magnetic field of given strength we have the means of obtaining a unit of resistance which is absolute—that is, it depends upon the same measures by which we estimate velocity in mechanics,

or is ultimately referred to the motion of a pendulum, and therefore to the attraction of gravitation.

We know from the preceding that we can also express the unit of electro-motive force in absolute measure; for suppose we have a rectilinear conductor of length l (Fig. 110), perpendicular to the lines of magnetic force, and moving in a uniform magnetic field of the intensity H, perpendicularly to the lines of force. Let this wire move along two parallel rails, $A B$ and $C D$, with a velocity v. By this motion we obtain an electro-motive force $E = l T v$, and, therefore, a current through $B G D$. This shows that our unit of electro-motive force can be taken as that force which would be induced in a rectilinear conductor of unit length, moving with a unit velocity across a field of unit magnetic strength, in a direction perpendicular to itself and to the lines of magnetic force.

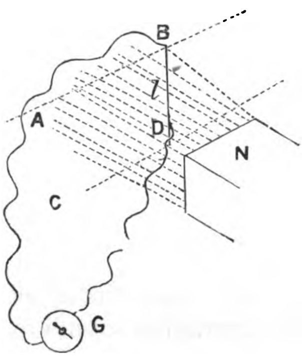

FIG. 110.

Thus we see that we can also base our electro-motive force upon what we have called absolute measure. If the intensity of the magnetic field is taken as one unit in the centimetre-gramme-second system of units, the distance between the rails $A B$ and $C D$ as 1 centimetre, and the velocity of the conductor l as 1 centimetre per second, the electro-motive force or difference of potential produced will be 1 centimetre-gramme-second unit of electro-motive force. This unit is

too small for practical purposes, and the difference of potential produced by such a slider or conductor moving with a velocity of 100,000,000 centimetres per second is taken as a practical unit, and is called a volt. It is very nearly the electro-motive force of a Daniell cell.

The work to produce an electro-motive force in a closed conductor when it is moved across the lines of magnetic force is very evident to the senses when the field of force is a strong one. It requires a steam-engine of several horse-power to revolve a coil in a strong magnetic field in order to produce the electric light. A consideration of the work done in producing a given current will enable us to understand how the units of electro-motive force and of current are based upon the same units as those we have adopted in mechanics and in heat. In the first place, let us see how far the doctrine of the conservation of energy can already be applied.

EXPERIMENT 130.—Connect the poles of a battery with an ordinary electro-magnet such as is used on telegraphic sounders. Make a small coil of very fine copper wire, similar to those used upon the magnet of the Bell telephone, and connect this small coil with an astatic galvanometer. On quickly bringing one face of this coil to one end of the electro-magnet, a current will be induced in it in one direction; on removing the small coil along the extended axis of the electro-magnet, a current will be induced in the opposite direction. We know that if the conservation of energy is true in regard to electro-magnetism, the direction of these induced currents should be such as to oppose our motion in both cases, for these currents in the small coil make it for

an instant a little electro-magnet having attracting and repelling poles. Suppose that, in moving the small coil toward the north pole of the large stationary electro-magnet, we should induce a current in the small coil in such direction as to make the end of the small coil toward the north pole of the stationary magnet a south pole; the attraction between these poles would then draw the small coil to the larger, and would aid our effort to move them together; at the same time we should gain a certain amount of heat from the current which has been induced in the small coil. Suppose we should fire a cannon-ball from a high elevation vertically downward. The energy with which the ball strikes the earth would depend upon the velocity with which it starts and the velocity gained under the attraction of the earth. The energy expended upon it in the shape of the combustion of powder has its equivalent in the work it can do with the velocity given to it. In the case of the small moving coil we exert a certain amount of energy upon it to give it a magnetic potential with respect to the attracting stationary magnet. The work that it does in falling, or in approaching the attracting magnet, not only is equivalent to the energy gained, but we have in addition the heat produced by the electrical induced current which has been produced from nothing. This is evidently impossible, and we must therefore conclude that the currents induced in the coil when it is moved toward the electro-magnet, and when it is moved away along the same path, are such as to form in the first case a repelling pole, and in the second case an attracting one. The direction of induced current, therefore, can always

be learned from the application of the doctrine of the conservation of energy.

Since we find that currents of electricity are produced in coils which are moved in a magnetic field, it is evident that we could make an apparatus by which, with the expenditure of mechanical work, we could produce strong currents of electricity. Suppose we should wind a ring of iron with wire continuously (Fig. 111), and then revolve this ring between the poles of a permanent horseshoe magnet, the plane of the ring being perpendicular to the axis of revolution, which in turn is perpendicular to the plane in which the poles of the permanent magnet are placed.

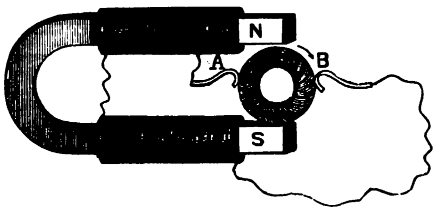

FIG. 111.

While one of the spires of the wire on the ring is approaching the north pole, a similar spire is approaching the south pole of the permanent magnet. We know, from experiment and from the preceding reasoning, that the currents induced in the wire of the ring will be opposite in direction, and will neutralize each other. Suppose, however, we should remove the covering of the wire along the periphery of the ring, and allow the brass spring to rub against this periphery at the points *A* and *B*. These springs, which are called commutators, will collect the opposite currents in the two halves of the ring, and the united currents will then flow from *A* through the outside circuit to *B*. On the way

to B, however, it is customary to wind this wire in equal coils upon the poles of the magnet between which the ring revolves. It will then be seen that the current produced by the motion of the ring can be made to strengthen the magnetism of the poles of the permanent magnet. The stronger the current becomes in the wire of the ring, the stronger becomes the magnetic field in which it revolves, and the more work it requires to turn the ring. This is the principle of the ordinary dynamo-electric machine which is used to produce the electric light. Instead of a permanent magnet, coils of wire with ordinary soft-iron cores are used; for the slightest amount of magnetism in the iron, together with the magnetic effect of the induced current produced by the revolution of the ring, is sufficient to start the machine. Its law of increase is, then, like that of compound interest. There is a certain limit, however, to its efficiency, for iron can not be magnetized to an indefinite amount. On estimating the work done in turning the ring, or, as it is called, the armature of a dynamo-machine, and comparing it with the heat produced in the circuit, we find, on estimating the losses that take place, that there is an exact equivalence between the two. If we call H the amount of heat evolved by the current S, in a conductor of resistance R, in the time t, we find $H = S^2 R t$. Eq. (1). The practical unit of electrical resistance when the length $R = \frac{2L^2 n}{K \tan\phi}$ (p. 234) is expressed in centimetres, is 10^9, or 1,000,000,000; having determined upon this, we can find the value that S must have in the equation $H = S^2 R t$, in order that the unit of work shall

be one gramme raised one metre, or 100 centimetres high.

We know that the unit of force adopted in mechanics is that force which in unit time will communicate a unit velocity to a unit mass.

The general expression for force is $F = Mv$. Since $v = \frac{\text{space}}{\text{time}} = \frac{s}{t}$, we have $F = \frac{Ms}{t}$.

To obtain the force in the unit of time, we must divide by t.

Hence, the unit of force $= \frac{Ms}{t^2}$.

The expression $\frac{Ms}{t^2}$ is called the dimensions of force. The force exerted on M grammes by the earth's attraction $= 980.6\ M$. If M is one gramme, our unit of force becomes in the centimetre-gramme-second system, 980.6 (latitude of Paris).

In the metre-gramme-second system it is 9.806. This, it will be remembered, is the same as g, the acceleration produced by gravitation. The value of the earth's force of gravitation and the acceleration imparted by it to one gramme are the same.

Work is performed when the point of application of the force is moved by it. Work is equal to the force multiplied by the distance through which it acts, or $W = Fs$. Since the value of $F = \frac{sM}{t^2}$, we have $W = \frac{Ms^2}{t^2}$.

To raise, for instance, one gramme one metre, we must do the work 9.806, since in this case $W = Fs = 9.806 \times 1 = 9.806$; or, if the centimetre-gramme-second system is used, $W = 980.6 \times 1$ ×

$100 = 980.6 \times 10^2$. The value of current, therefore, which, working through the unit of resistance $R = 10^9$ in one second, will correspond in units of work to nearly 102 grammes raised one metre high in one second, or 102 gramme-metres, will be $102 \times 980.6 \times 10^2 \doteq S^2 R t = S^2\, 10^9 \times 1$, or $S = \frac{1}{10}$. This is called an ampère.

We can, therefore, form the following expression: $S^2 R t c = h W \times 772$, or $S = \sqrt{\frac{W h 772}{R t c}}$, in which $W =$ weight of water in pounds, $h =$ increase of temperature in Fahrenheit degrees, $772 =$ mechanical equivalent of heat, $t =$ time in seconds, $c = .7373 =$ equivalent in foot-pounds of one ampère per ohm per second.

EXPERIMENT 131.—Measure the strength of an electrical current in ampères by running the current through a fine piece of iron or platinum wire, which is immersed in a given calorimeter such as is used in Experiment 88, which contains a given amount of water. Be careful that the wire does not touch the sides of the calorimeter.

Having decided upon the practical value of the unit of current S, we have from Ohm's law $S = \frac{E}{R}$, and $\frac{1}{10} = \frac{E}{10^9}$, or $E = 10^8$.

The unit of current $\frac{1}{10}$ in the centimetre-gramme-second system, CGS system, is called an ampère, from Ampère, the great French physicist. The unit of electro-motive force, $E = 10^8$, is called a volt, from Volta, the Italian physicist.

These units appear arbitrary at first, but the

unit of resistance being expressed and determined as a velocity, we can express our unit of current in the same decimal system, so that it may have as near as possible the value of 100 metre-grammes. It, moreover, happens that, taking the unit of resistance $= 10^9$ and the unit of current $\frac{1}{10}$, or 10^{-1}, which in turn give the unit of electro-motive force 10^8, a Daniell cell is found to have an electro-motive force of about one volt. The Daniell cell being the most constant cell which is in common use, this system adapts itself readily to practice. Let us now see how electro-motive force can be measured in these absolute units.

The electro-motive force of a cell does not depend upon the size of the metal plates of the battery.

EXPERIMENT 132. — Obtain the electro-motive force of two gravity-cells, in one of which the plate of zinc or iron is twice as large as in the other.

The internal resistance of the cell, however, is much modified by the size of the plates, for the resistance of a layer of liquid follows the same law as that of a metallic conductor, varying directly as the thickness of the layer and inversely as its section. The greater the size of the metallic plates of the battery, therefore, the less the resistance of the liquid between them. The operation of coupling batteries for quantity consists merely in reducing or increasing the battery resistance in comparison with that of the exterior circuit, by coupling a certain number of the zinc plates together and the same number of the carbon or copper plates together, so as to form larger plates, and therefore in-

creasing the section of the liquid and reducing the resistance of the liquid.

Fig. 103 shows four cells arranged for intensity; that is, so that both the electro-motive force and the battery resistance are increased, the strength of current being $S = \frac{4E}{4B + r}$.

Fig. 112 shows the four cells arranged for quantity. In this case we have $S^1 = \frac{2E}{2B + r}$.

EXPERIMENT 133.—To determine the electromotive force of a Bunsen cell in terms of a Daniell, connect the Daniell cell through a galvanometer and also through a certain resistance, R. Call the resistance of the galvanometer g; this can be measured by the Wheatstone bridge. If K is the reduction factor of the galvanometer employed, we shall have for two different values of the resistance R (B being the battery resistance) $S = \frac{E}{B+g+R} = K \tan \alpha$, $S_1 = \frac{E}{B+g+R_1} = K \tan \alpha_1$, or $\frac{B+g+R_1}{B+g+R} = \frac{\tan \alpha}{\tan \alpha_1}$. From this we can obtain B, the internal resistance of the battery.

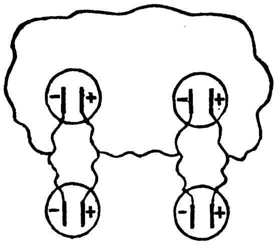

FIG. 112.

Obtain also the internal resistance of the Bunsen cell. Then compare the deflections produced by the two cells through known resistances (which include the battery resistance, the galvanometer re-

sistance, and the resistance R, interposed together with the connecting wires).

We shall then have $S = \frac{E}{B + g + R} = K \tan a$ for the Daniell cell, $S_1 = \frac{E_1}{B_1 + g + R_1} = K \tan a_1$ for the Bunsen cell, or $\frac{E}{E_1} = \frac{B + g + R}{B_1 + g + R_1} \frac{\tan a}{\tan a_1}$.

The deflections should not exceed 45°, for a greater deflection moves the needle out of a uniform magnetic field.

EXPERIMENT 134.—Another method is to connect the cells together and obtain their combined effect upon the galvanometer, or

$S + S_1 = \frac{E_1 + E}{B + B_1 + g + R} = K \tan a$, and afterward run them against each other and obtain the difference of their effects, or $S_1 - S = \frac{E_1 - E}{B + B_1 + g + R}$ $= K \tan a_1$, or $\frac{E_1 + E}{E_1 - E} = \frac{\tan a}{\tan a_1}$.

The denominator $B + B_1 + g + R$ is the total resistance of the circuit, which is the same for both cases, and therefore cancels out. E being the electro-motive force of the Daniell cell, E_1 can be obtained in terms of it.

This method forms a control method to that employed in the preceding experiment, but this method can only be regarded as approximative, since the stronger cell affects the weaker by what is called polarization, which will be explained later.

Perhaps the most instructive method of determining electro-motive force is that of Poggendorff. Suppose we should have two water-engines, E' be-

ing the stronger. E' takes in water from a reservoir and forces it through the pipe ABC back to the same reservoir. If a smaller engine, E (Fig. 113), endeavors to also force water into the pipe at B, its ability to do so will depend upon the pressure it can exert at B. The difference of level between B and C will be just equal to the pressure the engine E can maintain if no water flows through BEC.

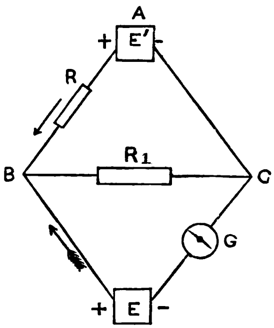

FIG. 113.

Substituting galvanic cells for our water-engines, and connecting the similar poles in the manner shown in the figure, by placing a galvanometer, G, in the circuit of the lesser cell, we shall find, by properly adjusting the resistances R and R_1, that no current will pass through the galvanometer G which is placed in the circuit of the weaker cell. The current through ABC will be $C = \frac{E'}{B_1 + R + R_1}$, B_1 being the resistance of the battery E'.

The current from B to C is the same as would be maintained by the battery E through the resistance R_1; hence we shall also have $C = \frac{E}{R_1}$, or

$$\frac{E^1}{B_1 + R + R_1} = \frac{E}{R_1}, \text{ or } \frac{E^1}{E} = \frac{B_1 + R + R_1}{R_1}.$$

This is an application of the general law (called Kirchhoff's law) that the current between any two points depends only upon the difference of potential or level maintained between its two ends, and also

upon the resistance of the conductor between these two points; or, calling the strength of the current C, we have $C = \frac{E}{R}$, or $CR = E$, E being the electro-motive force which is directly proportional to the difference of potential between A and B.

EXPERIMENT 135.—Ascertain by a galvanometer the truth of Kirchhoff's law for a circuit. Form a divided circuit at the point B and C (Fig. 114). The current will divide at B, and after flowing around both branches, $B\,G\,C$ and $B\,m\,C$, will unite again at C. The current C through $B\,G\,C$, plus that through $B\,m\,C$ or C_1, will be equal to the whole current S between A and B or $C + C_1 = S$. Placing a galvanometer, therefore, in one of the branches, $B\,G\,C$, another G_1 being placed in the circuit between A and B, or at G^1, we shall find that C multiplied by the resistance between B and C or $C\,(G + R_1)$ will be equal to the electro-motive force of the battery or $C\,(G + R_1) = E$. We conclude from symmetry that the same law must apply to the circuit $B\,m\,C$ or $C_1(R_2) = E$, R_2 being the resistance of $B\,m\,C$. Knowing, however, the strength of the current $S = C + C_1$, and having obtained C, we can show that Kirchhoff's law also holds for the circuit $B\,m\,C$. We therefore see that $\frac{C}{C_1} = \frac{R_2}{G + R_1}$, or the current divides inversely as the resistances of the divided circuit.

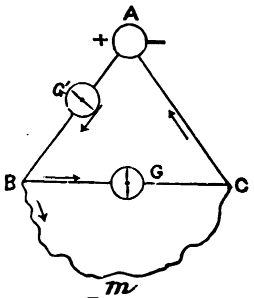

FIG. 114.

These methods of determining electro-motive force are relative, and not direct. Let us now examine more direct methods and obtain the exact value, as far as our apparatus permits, of the electro-motive force of a Daniell cell.

EXPERIMENT 136.—Pass the current from a Daniell cell through a known resistance, R, and a galvanometer, having obtained the resistance, B, of the cell, and G, that of the galvanometer. Knowing the reduction factor of the galvanometer $= \frac{a\,T}{2\,n\,\pi} = K$, we then have $S = \frac{E}{B+G+R} = K \tan a$, and $E \times 10^8 = (B+G+R) \times 10^9 \times K \tan a$, or $E = \frac{(B+G+R)\,10^9\,K \tan a}{10^8}$, 10^8 being the unit of electromotive force and 10^9 that of resistance.

Many of our experiments thus far do not require a knowledge of the reduction factor of the galvanometer. We have shown how this factor is obtained from the dimensions of the coil, and from the value of the earth's magnetic field. In certain cases it is difficult to obtain the dimensions of the coils. If we know the reduction factor of one galvanometer, we can readily obtain that of another in the following manner:

EXPERIMENT 137.—Pass the same current through two galvanometers, the reduction factor of one being C, and the other C^1. C being known, we shall then have from one instrument S = strength of current $= C \tan a$, and from the other $S = C^1 \tan {}^1a$. Hence we have $C \tan a = C^1 \tan a^1$, or $\frac{C}{C^1} = \frac{\tan {}^1a}{\tan a}$. Knowing C, we can obtain C_1.

EXPERIMENT 138.—To obtain the reduction factor of a galvanometer of which we can not obtain the

number of turns. Ascertain accurately the electromotive force of a Daniell cell, connect it with the galvanometer, and interpose a very large resistance in the circuit. We shall then have $S = \frac{E}{R} = C \tan \alpha$, or $C = \frac{E}{R \tan \alpha} = \frac{E \times 10^8}{R \times 10^9 \tan \alpha}$ in C. G. S. system.

We have thus far used the expression reduction factor for the number by which we must multiply the tangent of the deflection of a galvanometer to obtain the strength of the current. It is usual to separate this factor into two quantities, one of which is T, the horizontal intensity of the earth's magnetism, and the other is $\frac{2 \pi n}{a}$, which depends merely upon the number of turns of wire and the radius of the coil; this latter factor is always constant for the same instrument, and hence is called the constant of the galvanometer. The strength of a current will then be $S = \frac{T}{m} \tan \alpha$, in which $m = \frac{2 \pi n}{a}$.

Let us now return to the expression for the work done by any current of electricity. This is $H = S^2 R t$.

Instead of determining the value of S by means of a galvanometer, we can use the value of the electro-motive force, for $S = \frac{E}{R}$, and substituting this, we have $H = \frac{E^2 t}{R}$. By the use of an electrometer, we can determine E in terms of a standard Daniell cell, for the ratio of the deflections produced by the Daniell cell, and the cell whose electro-motive force is to be determined, is the same as the

ratio of their difference of potential, and this in turn is the ratio of the electro-motive forces.

EXPERIMENT 139.—To determine the electro-motive force of a battery or a dynamo-electric machine by comparison with one or a number of Daniell cells, we have merely to connect the terminals of the electrometer to the poles of the battery or to the terminals of the dynamo-machine, having interposed a sufficiently large resistance in the circuit between these poles. Since we must observe the deflections of the mirror of the electrometer by reflection of a spot of light upon a scale, having obtained the values of tan α for the cells which we wish to compare, we then have $\frac{E}{E_1} = \frac{\tan \alpha}{\tan \alpha_1}$, E being the electro-motive force of the standard cell.

In general, however, quantitative measurements with an electrometer are more difficult than with a galvanometer, on account of the care that must be taken to keep a constant charge on the needle of the electrometer. This can be done, however, with a water-battery. Instead of an electrometer, we can use a galvanometer of a very high resistance to determine electro-motive force. Such a galvanometer can be made very simply if the secondary coil of an induction-coil can be separated from its primary.

EXPERIMENT 140.—Cut a bottle in two, place a small bit of magnetized watch-spring upon the back of a small mirror, and hang it up by a single cocoon-fiber in the center of the lower half of the bottle; push this half-bottle, with the mirror suspended in it, into the axis of the secondary coil. Having mounted the coil horizontally on a firm support, a spot of light can then be used to observe the de-

flections of the magnet when a current is passed through the coil. If a plain mirror is used, the deflections can be observed with a telescope and scale. Discharge a Leyden-jar which has been only moderately charged through this galvanometer (there being no danger of destroying the insulation between the coils of this galvanometer). It will be found that this sudden discharge or fall of electrical potential will give evidence of an electrical current; the effect being the same as if we should send for an instant a current from a battery through the instrument. It is evident, therefore, that we can measure the quantity of electricity in a Leyden-jar by such an instrument.

In order to understand the effect of a sudden current in a coil upon a magnetic needle at its center, it is necessary to return to the subject of mechanics. We have seen how the work of a projectile can be estimated by the height a ballistic pendulum is lifted against the force of gravitation. We can liken the sudden creation of a field of force at the center of a coil, and the sudden impulse given to the magnet, to the circumstances of the blow and the effect of the projectile on the ballistic pendulum. Let us first examine our ideas of angular motion.

In the case of the ballistic pendulum, we have the bob of the pendulum moving through a certain arc with a certain velocity. The angle traced by the bob in one second measures this velocity and is called the angular velocity.

In general we define angular velocity as the angle traced in one second by a line fixed in the body and moving at right angles to the axis of rotation of the body.

Angular velocity can be uniform, as we have supposed in our first experiments upon the fall of bodies; or it can be variable. In the case of the ballistic pendulum it is variable.

If ω is the angular velocity of a point on the arc of a circle whose radius is one, then the arc traced by the point in one second measures the angular velocity. Any other point on the continuation of this radius at a distance, r, from the center of the circle will describe an arc, $r\omega$, in the same time that the arc ω is described.

The energy of a body rotating with the angular velocity ω is $\frac{1}{2} m r^2 \omega^2$, $r\omega$ being substituted for v in the expression $\frac{m v^2}{2}$. We perceive that $m r^2$ is the expression we have called moment of inertia in our discussion of the compound pendulum. We can therefore write the above expression $\frac{1}{2} K \omega^2$, in which K is the moment of inertia.

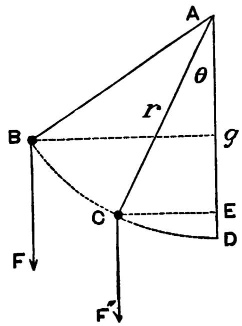

FIG. 115.

In the case of the pendulum acted upon by gravitation, the pendulum has a certain angular velocity ω at D (Fig. 115) and another angular velocity ω_1 at C and no angular velocity at B, the highest point of its swing. Hence, if we call F the force acting upon the pendulum-bob at a distance of r from the point of suspension A, we have the work done in lifting the bob from D to C through the height $DE = F\,DE = Fr\,(1 - \cos\theta)$.

The energy at $D = \frac{1}{2} K \omega^2$ and that at $C = \frac{1}{2} K \omega_1^2$; hence the work between D and C will be $Fr(1 - \cos\theta) = \frac{1}{2} K(\omega^2 - \omega_1^2)$.

If we call the angle $BAD = a$, we shall have, when the bob reaches its highest point B, $\omega_1 = 0$, or $Fr(1 - \cos a) = \frac{1}{2} K \omega^2$.

From a table of trigonometrical relations we find that $1 - \cos a = 2 \sin^2 \frac{a}{2}$. Hence, $\omega = 2 \sin \frac{a}{2} \sqrt{\frac{Fr}{K}}$.

In the case of magnetism, the force acting upon the magnetic pendulum is $\mu H r$ instead of Fr; the strength of the pole being μ, that of the field being H.

If we call μr the magnetic moment, and denote it by G, we shall have in the case of the magnetic pendulum $\frac{K\omega^2}{2} = HG(1 - \cos a)$,

or, $\omega = 2 \sin \frac{a}{2} \sqrt{\frac{GH}{K}}$.

The work done in turning a body through a small angle, θ, is the force multiplied by the distance passed over, or the arc. Hence,

$$\text{work} = F \times l\theta.$$

If ω and ω_1 are the angular velocities at the beginning and end of the movement, we have the energy expressed by

$$\tfrac{1}{2} K(\omega^2 - \omega_1^2) = F \times l\theta = F l \omega_1 \tau,$$

in which τ is the time during which the force acts,

$$\tfrac{1}{2}(\omega^2 - \omega_1^2) = \tfrac{1}{2}(\omega - \omega_1)(\omega + \omega_1)$$

$$\tfrac{1}{2}(\omega + \omega_1) = \omega_1, \text{ approximately.}$$

Hence, $K(\omega - \omega_1) = Fl\tau$.

In the case of a galvanometer we have

$$F = \frac{2\pi n i \mu}{a} \text{ (} i \text{ being strength of current).}$$

If we denote $\frac{2\pi n}{a}$ by C, and μl by G, we have $K(\omega - \omega_1) = C G i \tau$, but $i\tau = q =$ quantity of electricity transmitted. Hence, $K(\omega - \omega_1) = C G q$.

If the magnet moves through a very small angle (and this can be accomplished if the discharge through the coil takes place very quickly), we can suppose that the force is always perpendicular to the magnet, and can take $K\omega = C G Q$, in which Q is the whole quantity of electricity transmitted; but we have $\omega = 2 \sin \frac{a}{2} \sqrt{\frac{G H}{K}}$. Hence,

$$K\omega = 2 K \sin \frac{a}{2} \sqrt{\frac{G H}{K}} = C G Q,$$

$$\text{and } Q = 2 \frac{H}{C} \sqrt{\frac{K}{H G}} \sin \frac{a}{2}.$$

If T is the time of vibration of the magnet, we have found previously, $T = \pi \sqrt{\frac{K}{H G}}$.
Hence, $Q = \frac{2 H}{C} \frac{T}{\pi} \sin \frac{a}{2}$.

We always have $Q = S t$, where S is the current and t is the time; or, $Q = \frac{E}{R} t = \frac{2 H}{C} \frac{T}{\pi} \sin \frac{a}{2}$.

From this expression we can obtain the electromotive force if we know the time during which the discharge lasts, or we can obtain the ratio between two electro-motive forces if we make the times of discharge equal, for τ will then cancel out in the ratio of the electro-motive forces.

With the aid of a galvanometer of high resistance, we can obtain the electro-motive force of a series of batteries or of a dynamo-machine. It is

often important to be able to do this, for the accurate measurement of current depends upon the comparatively small deflection of a magnetic needle in a uniform magnetic field, and these conditions are not always easily and accurately obtained. For instance, a strong current from an electrical machine, if passed through a galvanometer of one coil, will ordinarily give a deflection of more than 45°. In this case the field usually ceases to be uniform, and the deflections are not accurately proportional to the tangents of the angles. If we proceed to shunt the galvanometer—that is, to direct a part of the current through a wire of resistance *m* (Fig. 114), connecting the binding-screws of the galvanometer—we shall find that even a small error in the determination of the amount of wire we bind into the binding-screws of the instrument, will make a large error in our determination of the current.

For we have $\frac{m}{G} = \frac{C}{C_1} = \frac{K \tan a}{C_1}$, or, $C_1 = \frac{GK \tan a}{m}$,
where G is the resistance of the galvanometer and C and C_1 are the portions of the current passing through the two branches of the divided circuit.

Since we have also

$$S = C + C_1 = \frac{Km \tan a + GK \tan a}{m},$$

or, $S = K \tan a \frac{(m + G)}{m}$,

we see that a small error in the determination of m affects the whole value of S, G being usually very small, and R being less than G. It is, however, difficult to obtain an accurate value of a small resistance.

With a high resistance galvanometer we can determine the electro-motive force of a large battery

or dynamo-machine by connecting one terminal of the galvanometer with one pole of the machine, and touching the other for an instant only to the other pole. In this case the sudden swings of the galvanometer (or, more accurately, the sines of one half the angles) are proportional to the difference of potential of these poles. Do the same with a series of Daniell cells upon a circuit of known resistance. We shall then have $\frac{E}{E_1} = \frac{\sin \frac{1}{2} \theta}{\sin \frac{1}{2} \theta_1}$, in which E is the desired electro-motive force, and E_1 that of the standard or Daniell cells.

Let us now make a measurement of the work done in producing a current of electricity by means of a dynamo-machine. Although these machines are usually run by a steam-engine, we will suppose that we have one which is worked by two men. The small dynamo-machine is run by a band which passes over a large fly-wheel which is turned by the men. A friction-brake is put upon the axle of this large wheel, and the work done in turning this with a certain velocity is measured. The electrical current, in the first place, is passed through a galvanometer of a known resistance in order to determine by the steadiness of its deflection whether the current is constant or not. The difference of potential between the binding-screws or poles of the machine is then measured by comparison with a certain number of standard cells whose electro-motive force has been determined. The work done by the electrical current will then be in metre-grammes,

$$\frac{\frac{E^2 R t}{R^2}}{980.6} = \frac{E^2 t}{R \times 980.6} = \frac{(10^8 E)^2 t}{R \times 10^9 \times 980.6} = W.$$

If we call the work done in turning the fly-wheel, expressed in metre-grammes, W_1, the efficiency of the dynamo-electric machine will be $\frac{W}{W_1} = A$.

It is supposed that the belt connecting the revolving axles does not slip. The same method can of course be applied to the case where a steam-engine turns the dynamo-electric machine.

Instead of measuring the current by the indirect method, suppose we adopt the direct method and use the deflection of a galvanometer. In this case it will be necessary to shunt the galvanometer, and the whole current will be $S = K \tan \alpha \cdot \frac{(R+G)}{R}$. The whole resistance of the circuit will include that of the battery or dynamo-machine, and the resistance outside or interpolar resistance. This interpolar resistance is made up of the connecting wires, of whatever resistance is interposed, and that of the shunted galvanometer. The use of a shunt alters the resistance of an instrument, as can readily be seen by measuring the divided circuit, as a whole, by means of a Wheatstone bridge. By considering the difference of potential between the two points A and B (Fig. 116), we shall see that some wire of resistance χ = combined resistance of the two wires, R and R_1, can be substituted between A and B, and the difference of potential or level $AD - BD^1$ can be maintained. Hence this difference of potential or

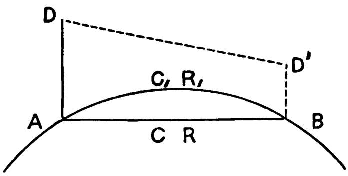

FIG. 116.

electro-motive force must be equal to the resistance χ multiplied by the current S, or $C+C_1$.

Now, we know that $\frac{R}{R_1}=\frac{C_1}{C}$, which is the same ratio as

$$\frac{R+R_1}{R_1}=\frac{C_1+C}{C}=\frac{S}{C}=\frac{\frac{E}{\chi}}{\frac{E}{R}}=\frac{R}{\chi}.$$

Hence the combined resistance $\chi=\frac{RR_1}{R+R}$. We shall then have W = work in metre-grammes, $=\frac{S^2 R_0 t}{980.6}=\frac{(S\frac{1}{10})^2 R_0 10^9 t}{980.6}$, R_0 being the entire resistance of the circuit, and the efficiency can be obtained as before. This method is open to the objection of the error produced by shunting.

If a battery is employed instead of a dynamo-machine, the process of obtaining the work it does in heat-units has already been described. The determination of the internal resistance of the battery should in general be made unnecessary by the arrangement of the experiment, for it is a very uncertain quantity. A simple method has already been given by which it can be measured. The following method is often useful, and is given to illustrate the simple laws of electricity which we have used in the preceding experiments:

EXPERIMENT 141.—To obtain the internal resistance of a battery, or a single cell, pass the current through a galvanometer and a known resistance, and note the deflection of the galvanometer. Call this deflection θ. We then have $\frac{E}{B+R+G}=K\tan\theta$, in which B is the resistance of the battery, G that of

the galvanometer, and R is the interposed resistance. Now shunt the battery by a wire of known resistance, M; in order to produce the same deflection as before, it will be necessary to change the resistance R to some other resistance, R_1.

We shall then have $\frac{M}{G+R_1}=\frac{C}{C_1}$, C and C_1 being the strengths of currents through $G + R_1$ and M, respectively. The whole current $S = C + C_1$, and by the conditions of the experiment $C = \frac{E}{B+R+G}$. Hence,

$$\frac{E}{B+\frac{M(R_1+G)}{M+(R_1+G)}}=\frac{E}{B+R+G}+\frac{G+R_1}{M}\,\frac{E}{B+R+G}$$

Solving for B, we obtain $B = M\left(\frac{R-R_1}{R_1+G}\right)$.

This method is open to the objection that any change of resistance in the outer circuit necessarily modifies the electro-motive force and internal resistance of the battery. It is difficult to avoid this by any method.

Instead of a battery, or of a dynamo-machine, we can employ the ordinary Holtz electrical machine to obtain the mechanical equivalent of heat. This machine can be likened to a water-battery of a very great number of cells. The internal resistance is very great, and the difference of potential maintained by mechanical work is also very great. Accurate measurements of the electrical work done by the electrical machine are difficult on account of the tendency of electricity of high potential to escape from conductors. The method employed, however, is extremely instructive, since it leads us to connect

the manifestations of electricity of high tension, such as is produced by the common electrical machines, with the manifestation of electrical potential exhibited by batteries. The Holtz electrical machine is run by some small motor, such as a water-engine. The mechanical work consumed can be estimated by some of the methods already given. The poles of the electrical machine are connected by wires to the terminals of a galvanometer of high resistance and of great insulation.

In order to obtain the reduction factor of our galvanometer, since we can not in general measure with accuracy the mean radius of the coil, or count the number of turns, we make use of a Daniell cell, knowing the electro-motive force of this cell. We shall have $\frac{E}{R} = K \tan \alpha$. Making R very large, and including in it the large resistance of the galvanometer, we can neglect the resistance of the Daniell cell. We shall then have $K = \frac{E\,10^8}{10^9 R \tan \alpha}$, in which K is expressed in *C. G. S.* units. If we then call S the current produced by a definite amount of work performed upon the Holtz machine, we shall have $S^2 R t$ = heat equivalent, $= J H$ = work equivalent. From this expression we can obtain J, the mechanical equivalent of heat.

We have thus far measured the magnetic and heating effects of the current. There are other phenomena, which have important bearings upon the doctrine of work, especially of molecular work. Among these phenomena are the effects of the electrical current in decomposing certain liquids. If we should attempt to measure the resistance of

a column of water by inclosing it in a tube, dipping platinum wires into the water at the extremities of this tube and connecting them with Wheatstone's bridge, we should obtain very inconstant results. This is due to the fact that the electrical current, in passing through the liquid, decomposes it, producing hydrogen at one of the electrodes or wires dipping into it, and oxygen at the other. These layers of gas, in turn, produce a difference of potential, as if another battery were interposed in the circuit, and the law of the Wheatstone bridge is no longer maintained. It would evidently be impossible to measure the internal resistance of a cell by putting it into one of the branches of a Wheatstone bridge, for, by constructing a diagram of the fall of potential, it would be seen that the proportionality between the adjacent sides of the bridge exists no longer, and the simple law of the bridge is destroyed. All liquids which can be decomposed by the electrical current are called electrolytes, and all metals immersed in them, conveying an electrical current from one point to another, become polarized—that is, are made + or − to each other. This polarizing effect is almost nothing when the electrodes conveying the current to and from the liquid form the base of the liquid; that is, when the liquid is a salt of these metals. For instance, zinc electrodes in sulphate of zinc, or copper electrodes in sulphate of copper, exert very little of this polarizing effect. It would be impossible, however, to measure the electrical resistance even of these non-polarizable liquids by introducing them into the Wheatstone bridge, for the current decomposes them—depositing the metal upon one electrode, the negative electrode.

The effect of the polarization of an electrical current can be shown in a very striking way by means of what is called a storage-battery.

EXPERIMENT 142.—Place two strips of sheet-lead, about six by eight centimetres, at a distance of six centimetres from each other, in a vessel containing water acidulated with sulphuric acid—one part of acid to ten of water. Pass the current from one Bunsen cell through the liquid from one plate to the other, placing also a low-resistance galvanometer in the circuit. It will be found that the deflection of the galvanometer grows less and less, and finally becomes steady. When this latter point has been reached, disconnect the battery and connect the lead plates with the galvanometer. A current will be obtained which is opposite to that of the battery. This is a polarization current, and is due to generation of oxygen on one of the lead plates and of hydrogen on the other, thus producing a difference of potential. It is evident, from the conservation of energy, that the resultant current must be opposed to that which produced it. Find the relation between the strength of the current of polarization and that of the current which produces it.

The lead cell we have described is called a Planté cell, from the name of its inventor. The Faure cell consists merely of lead plates covered with a layer of red oxide of lead, which increases the polarization. It is evident that electricity is not stored in these cells. A difference of chemical constitution is produced, which in turn gives a difference of electrical level.

EXPERIMENT 143.—Pass a current through a galvanometer and through two copper strips immersed

at a fixed distance from each other in a bath of saturated solution of sulphate of copper. Allow the current to run for a given length of time, and then weigh the negative electrode. Do the same with a different strength of current (obtained by altering the resistance of the circuit). It will be found that the amounts of the copper deposited in the same time will be proportional to the current strengths.

If W and W_1 are the weights of copper deposited, and S and S_1 are the currents, we have $\frac{W}{W_1} = \frac{S}{S_1}$. Hence, the amounts being proportional to the time the current runs, $\frac{W}{t} = K \tan a$ will express the strength of the current in the amount of copper deposited in one second. It has been found that a current of one ampère ($S = \frac{1}{10}$) will deposit .01972 of a gramme of copper in one minute, or .0033 of a gramme in one second.

EXPERIMENT 144.—Determine the horizontal intensity of the earth's magnetism by means of the deposition of copper. This method is not considered so accurate as the method previously given, but it is often employed as a control method. Knowing the constant of a galvanometer, pass the current through it and a suitable vessel which contains the solution of copper. The current will vary during the experiment on account of the increase of resistance in the liquid produced by the deposition of copper. It is best to observe the deflections of the galvanometer at regular intervals, and to take the mean of the deflections. The angle of deflection should not exceed 45°. A simple method of forming a voltameter (the term applied to the vessel

filled with the electrolyte) is to replace the copper pole of an ordinary Daniell cell by a spiral of copper wire, which is hung in the saturated solution of sulphate of copper. In this case the battery itself constitutes the voltameter. We shall have $\frac{W}{.0033t}=\frac{r\,T}{2\,\pi\,n}\tan\alpha$. This will give us the current in ampères, and by solving we can obtain the factor T. The electrode upon which the copper is deposited should be carefully washed and dried before weighing. Table VII, Appendix, enables us to reduce the values we obtain from the use of an electrolyte to absolute measure, or magnetic measure. If we should collect the gases formed by the decomposition of water, we should find that the volume of hydrogen would be twice that of oxygen. Finding the amount of water decomposed by one ampère, we could also use this method for determining the value of T; or, knowing T, to determine the value of the reduction factor of the galvanometer. Our measurement, however, would be inaccurate, owing to the absorption of the gases in the water over which they are collected. This method is not often employed in a laboratory, because it requires a comparatively strong current—from two to four Bunsen cells.

CHAPTER XII.

THERMO-ELECTRICITY.

THE experiments which we have given thus far show how the value of electrical work can be obtained in terms of mechanical work and illustrate the conservation of energy. Let us not lose sight of our effort to connect all the manifestations of motion, heat, electricity, magnetism, and light together, as illustrating the doctrine of the conservation of energy. We have treated the galvanic cell merely as a motor which maintains a difference of level or difference of potential. We have perceived by direct experiment that any two metals immersed in an electrolyte will give a difference of electrical level, or potential. This difference of electrical potential is capable of doing work, as all difference of potential is capable of doing. We have measured this work in heat and then in mechanical units; and we have shown that the ordinary electrical machine can be likened to a galvanic battery made of a very great number of cells with large internal resistances. It is not necessary to know whether the electrical state is one of molecular motion or is due to the flow of fluids, in order to study the subject from our present point of view. We have occupied ourselves merely with the question of the

work done by a fall in potential, and we have studied electricity in the same manner and with the same units that we have employed in the study of mechanics.

Let us pursue the subject from the same point of view somewhat further. We have seen that any two different metals will, on being placed in contact or in an electrolyte, exhibit a difference of potential. We shall find also that the junction of two different metals, on being heated, will also show a difference of electrical level or potential.

EXPERIMENT 145.—Solder a piece of German-silver to each end of a piece of iron wire. Connect the ends of the German-silver wire to the terminals of an astatic galvanometer. Place one of the junctions of wire and German-silver in melting ice or in water of a known temperature, and heat the other junction by placing it in a beaker of water and heating the beaker gradually. Place a thermometer also in the latter beaker. It will be found that the deflections of the galvanometer indicate an electrical current, which arises from the difference of heat potential which is maintained between the junctions of iron and German-silver. If we lay off the degrees with a suitable unit of temperature indicated by the thermometer on a horizontal line, OX, and the deflections of the galvanometer parallel to the line OY, we shall find that the rise in temperature is uniform for small ranges of temperature—in other words, a straight line represents the rise. This indication of a uniform rise in temperature affords us a method of testing the accuracy of graduation of an ordinary mercury thermometer. To do this, obtain the deflection when one junction is at 0°,

or the temperature of melting ice, and the other at that of steam; and draw the line $O B$—the abscissa $O D$ representing 100° on the centigrade scale. Any intermediate temperature, $O C$, can be obtained by comparing the indication of the mercurial thermometer when the strength of current indicated by $B' C$ is obtained.

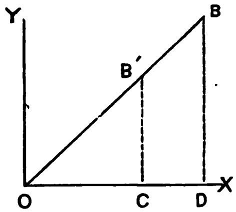

FIG. 117.

It is found that two different metals are not needed to produce a thermo-electric current. Any difference of temperature, even in a circuit of one metal, will manifest itself by an electrical current.

EXPERIMENT 146.—Connect a piece of iron wire with the terminals of a galvanometer, and heat it at its middle point: a thermo-electric current will be produced. Here the two junctions of iron with the terminals of the galvanometer are at the same temperature, that of the room, and the current is produced by a difference of temperature between different portions of the iron wire. Any change in structure, such as is produced by twisting or bending, will also produce an electrical current. This thermal electrical current is generally regarded as the evidence of a difference of potential created by different rates of internal changes among the molecules of the different metals. Instead of the mercury thermometer which we employed in our heat experiments, a junction of iron and German-silver wire can be employed, one junction being immersed in melting ice and the other in the liquid whose temperature we wish to measure. It is best to com-

pare the indications of the galvanometer with those of a standard mercury thermometer for certain temperatures; for the indications of the thermo-electric junctions are subject to sudden fluctuations, which have to be carefully guarded against. By the use of a thermo-electrical junction, the experiment to test the law $H = S^2 R t$ can be conducted with the use of a comparatively small battery.

All the experiments on heat which have been given should now be repeated, a thermo-electric junction replacing the mercury thermometer. For this purpose a sufficiently sensitive astatic galvanometer should be made. The coil for a galvanometer of very low resistance can be made by cutting a long strip, about five millimetres wide, from a thin sheet of copper or brass, and coiling it into a small flat spiral, interposing oiled paper between the spires, and soldering thick connecting wires to the ends of the spiral. Suspend in front of this spiral, which is mounted vertically on the end of a box, an astatic combination of needles provided with a little plane or concave mirror. If a plane mirror is used, a lens of long focus should be placed directly in front of it, in the manner we have already described. The lower magnet of the astatic combination should be made from a piece of watch-spring, and should be one centimetre long and two millimetres wide. This magnet is placed very close to the center of the spiral, leaving it, however, sufficient space to turn slightly. The magnet with opposite poles should be placed near the periphery of the spiral, directly above the lower magnet, and rigidly connected with it by a light, stiff filament of

glass (obtained by melting a glass tube and quickly drawing it out into a narrow thread). Thick liquid shellac is useful for connecting the magnets to the glass filament. The suspension fiber should be ten centimetres long. The entire galvanometer can be inclosed in another box provided with a glass end. This instrument can be easily made to measure one fiftieth of a centigrade degree.

The use of the thermo-electric element in measuring heat leads us to the subject of the distribution of energy in light of different colors, and thus leads us indirectly to the application of the doctrine of the conservation of energy to the subject of light.

By increasing the number of junctions of two different metals, and maintaining one set of junctions at one temperature—that of melting ice—and submitting the other corresponding set to the temperature we wish to measure, we can greatly increase the sensitiveness of the arrangement, which we then call a thermopile. It is found that junctions of bismuth and antimony give a comparatively large electro-motive force.

EXPERIMENT 147.—To obtain the efficiency of a thermopile, we must measure the amount of fuel in heat units we expend in producing a measured strength of current, and then, by the application of the law $J H = S^2 R t$, obtain the value of the current in units of work. If the thermopile is heated by a gas-burner, we must measure the amount of gas burned, and compute its value in heat units.

CHAPTER XIII.

SOUND.

THE close relationship of light and heat to electricity having been seen, it is simpler to begin our study of wave-motion by means of the phenomena of sound; and afterward, through the mechanical conceptions that we have thus gained, to turn to light as another manifestation of wave-motion.

Let us in the first place examine the motion of a tuning-fork. When we press together the prongs of the fork by means of the fingers, and suddenly release them, the prongs spring back not only to their original position, but to a point beyond, and continue to vibrate to and fro, as we have seen in the experiments in mechanics, in which we have used the tuning-fork as a pendulum. The time that is occupied in swinging from one side to the other, and then back again, is called the periodic time, or the period of vibration; and the distance that any point on the prong of the fork travels in executing a vibration is called the amplitude of vibration for this point.

Sound is a sensation in the ear, produced by the vibratory motion of bodies upon some elastic medium which transmits the vibration to the tympanum or membrane of the ear. The medium of

communication is usually the air. A person under water, however, can hear sounds transmitted through the water. Sounds can be heard farther through water than through air. The more elastic the medium, the swifter is the propagation of sound. A vacuum does not transmit sound.

EXPERIMENT 148.—Attach a little toy bell to the rubber cork of Experiment 20, and, having expelled the air by steam, incline the flask in order to ring the bell. It will be found that the sound is very faint. On letting in air, the ringing will be plainly heard.

EXPERIMENT 149.—Compare two tuning-forks. Clamp the two tuning-forks in two vises side by side. Provide them both with little pointers. Smoke a piece of glass and place it upon a horizontal support beneath the prongs of the tuning-forks. Adjust the forks in the vises until the pointers just touch the smoked glass. Then, having set the tuning-forks in vibration with a violoncello-bow, draw the smoked glass rapidly away from the pointers and along the horizontal support. To do this uniformly, the glass should have a narrow strip of wood glued or cemented to one side, and this wooden strip should be pressed against the edge of the board which forms the horizontal support.

This experiment will show that the intensity or pitch of the tuning-fork depends upon the number of vibrations it makes per second. It will be found that the number of vibrations of the lower note are less than those of the higher note.

EXPERIMENT 150.—Compare the vibrations of a wire with those of a tuning-fork. Stretch a steel wire between two points of a horizontal support,

A B (Fig. 118). Attach the wire rigidly to one of its supports, *A*, and weight the wire at its other end, *B*, by allowing the wire to pass over a little cylinder and allowing the stretching weights to hang vertically. Place a little pointer at the middle of the

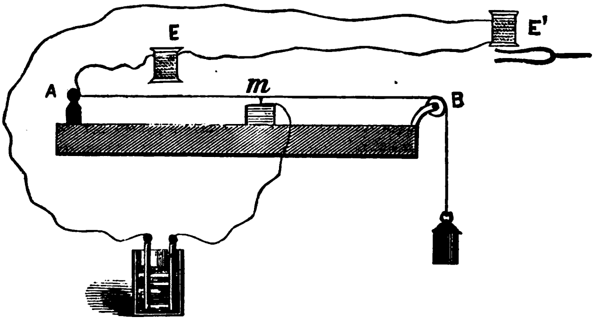

FIG. 118.

wire, *m*, and allow it to just touch the surface of mercury in the little cup below it. Place an electro-magnet, *E*, above the middle of the half-length of the steel wire. The wire will then vibrate. Place another electro-magnet, E^1, in the circuit, and ascertain what length of wire *A B* will cause a tuning-fork, *F*, placed very near the end of the electro-magnet E^1, to respond to the wire. It is well to cover the surface of the mercury-cup with a little alcohol and glycerine, to keep the surface clean.

EXPERIMENT 151.—Make one tuning-fork respond to another. Provide the prong of one tuning-fork with a little pointer. Allow this prong to just touch the surface of mercury in a little dish. Place an electro-magnet above the other prong, and arrange the electrical circuit so that it shall be

closed when the pointer attached to the prong touches the surface of the mercury. The tuning-fork will then be attracted by the electro-magnet, and the circuit will be broken, and the elasticity of the steel will quickly cause it to be made again. In this way the vibration of the tuning-fork can be maintained. Now arrange the second tuning-fork with another electro-magnet, placed in the same electrical circuit as the first electro-magnet. If the two tuning-forks give the same note, they will respond to each other.

EXPERIMENT 152.—Comparison of the vibration of a tuning-fork with the vibration of a straight rod. Clamp a tuning-fork beside a straight steel or wooden rod. Provide both the fork and the rod with metallic pointers, and adjust these pointers so that they shall just rest upon a plate of smoked glass. Set both the tuning-fork and the rod into vibration, and draw the smoked glass away. Compare the vibrations produced when the length of the rod is varied, and ascertain the law of the vibration of straight rods. Repeat the experiment with a rod of different section, and in this way study the influence of the section upon the rate of vibration. It is evident that the rate of the tuning-fork need not be known, since it is used relatively.

EXPERIMENT 153.—Investigate the laws of the vibration of strings by means of a tuning-fork. Clamp a tuning-fork by means of a vise in a horizontal position, and arrange the prongs so that the vibrations of the fork shall take place in a vertical plane. Attach a silk thread to one prong of the tuning-fork and a little tin cylinder to the other end of the string. Fill this cylinder with water, and

place it in a larger jar filled with water. By drawing off the water from the larger jar, either by a siphon or a stop-cock, it is evident that the tension on the string can be varied at will and by known amounts. On exciting the fork with a bow, the string will divide up into nodes and segments. Vary the length of the string and also the tension upon it, and ascertain its laws of vibration.

EXPERIMENT 154.—Obtain the time of vibration of a tuning-fork. Adjust the tuning-fork so that the pointer with which one of its prongs is provided shall touch the blackened surface of a revolving cylinder. Arrange the pointer attached to a pendulum-bob so that when the pendulum swings the pointer shall also draw a line upon the blackened cylinder. Excite the tuning-fork and set the pendulum in motion, and count the number of vibrations of the tuning-fork between the swings of the pendulum as they are indicated by the breaks in the straight line drawn on the cylinder by the pointer of the pendulum. Knowing the time of swing of the pendulum, or the interval between the breaks indicated by the pointer, we can obtain the number of vibrations of the tuning-fork in a second. The following apparatus will be necessary:

Obtain a wooden cylinder which is turned true about an iron rod that serves as its axis. The diameter of the cylinder should be about one foot, its length six inches. Two holes, about four inches deep and two inches wide, should be bored in the cylinder, parallel to its axis, and filled with lead. The iron axis of the cylinder should turn upon well-oiled supports. A little handle screwed upon one end of the cylinder, perpendicular to its axis, will

enable one to give a rapid motion to the cylinder. The cylinder should be covered with a sheet of well-glazed writing-paper, and then revolved gently over a smoking kerosene-lamp. On a stand placed at the side of the cylinder, and across the top of the box on which the axle of the cylinder rests, should be screwed a little iron vise, in which the tuning-fork can be adjusted.

EXPERIMENT 155.—Find the rate of a given

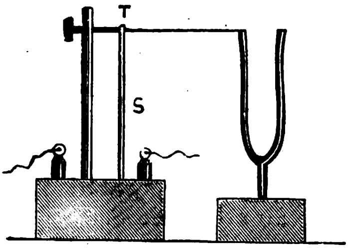

FIG. 119.

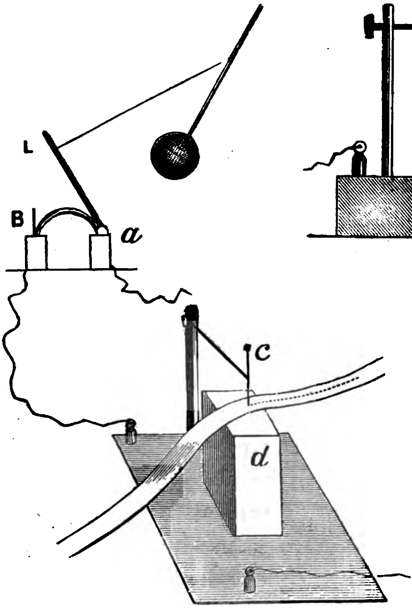

FIG. 120.

fork.* A silk fiber stretches from the prong of the fork to a slender vertical spring, *s* (Fig. 119), fixed at its lower end, while its upper end rests against a set screw, *t*. The two surfaces in contact are platinized. Every vibration of the fork is transmitted to the fiber, which causes the spring to open and close an electrical

* Professor Le Roy Cooley, "Journal of the Franklin Institute," vol. lxxii.

circuit in unison with the fork. Place this circuit-breaker in the same circuit with a heavy pendulum which swings about twice a second. Arrange this pendulum so that it closes the circuit during the time of a complete swing. This can be done as is shown in Fig. 120. *L a B* is a bent lever of metal, one arm of which is connected by a light fiber of silk with the pendulum-rod, and the other end, *B*, just rests upon a surface of mercury when the pendulum is at rest.

Arrange in another part of the circuit a flexible wire, *c*, which closes the circuit at *d*. Draw a strip of paper, moistened with a mixture of potassium iodide and starch, beneath this wire. While the circuit is closed by the bent lever connected with the pendulum, a blue line is drawn on the paper. If, however, the tuning-fork is vibrating, this blue line will be broken up into dots, and the number of dots gives the number of vibrations of the fork during the time of swing of the pendulum. It is best to have a little piece of platinum-foil beneath the strip of paper on the surface *d*. It is evident that the rate of movement of the paper must be such as to separate the dots so that they can be counted.

This method can also be used to investigate the transverse vibrations of wires.

EXPERIMENT 156.—Instead of a tuning-fork, use the apparatus of Experiment 150, and prove the following laws:

1. The number of vibrations in a given time varies inversely as the lengths of the wire.

2. Their number varies also as the square root of the tension of the wire.

In our study of the movement of pendulums we

have considered a vibration as the movement of the pendulum-bob from its highest position on one side of a vertical line to its highest position on the other side of this line. In discussions of wave-motion, a complete vibration is double this. Certain preliminary conceptions of harmonic motion are necessary to our study of sound.

It will be noticed that the point A (Fig. 121) on a rolling wheel draws a sinuous curve; and, moreover,

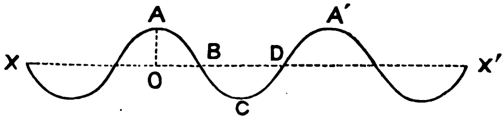

FIG. 121.

the distance between one summit, A, and the next, A^1, is divided into parts $A\ B$, $B\ C$, $C\ D$, $D\ A^1$, which are equal and are reversed in position. The distance $A\ O$ is the amplitude of the vibration, as can be seen by reference to the motion in a circle, and the distance traveled along the line $X\ X^1$, while the point on the wheel has moved over the entire circumference, is called a wave-length; thus, the distance $A\ A^1$ represents a wave-length.

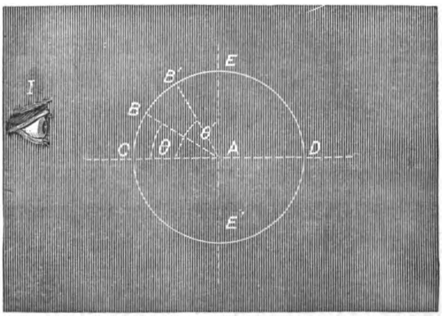

FIG. 122.

The motion of particles on a circle illustrates the expression "difference of phase." The position of the point B at the end of the radius $A B$ (Fig. 122) can be fixed by the angle it makes with a fixed line, $C A$. The points B and B' make different angles, θ and θ', with $C A$. The difference of phase is therefore proportional to $\theta - \theta'$. When the particles B and B' pass through E at the same time, the difference of phase becomes nothing. When one particle passes through E while the other is passing through E', the difference of phase is 180°. The period of a vibration is the time of moving from C completely around the circle to C again. If t represents the time of describing the angle θ, and T the period, we have $\frac{\theta}{2\pi} = \frac{t}{T}$. To an eye, I, placed in the plane of a vertical circle upon which B moves, the particle B appears to move up and down a vertical diameter $E E'$.

Suppose that the lines $a a_1$, $b b_1$, etc., represent equal parallel lines, and that we have simply vibration of particles along the lines $a a_1$, $b b_1$, etc. (Fig. 123). The position of any particle at any time with respect to any quadrant of the circle whose radius represents the amplitude of the vibration is called the phase. The particle r and particle s have a difference of phase, since, on the supposition that both are moving from left to right, the particle r will pass through E from the quadrant $C E$ to $E D$ (Fig. 122)

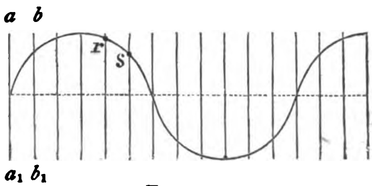

FIG. 123.

later than the particle s. If the periods of vibration of the particles are all equal, and the difference of phases also all equal, we have a simple harmonic undulation. The wave-length may be stated as the horizontal distance between one particle and the next when they are in the same phase. A particle at the highest part of its vibration is at the crest of a wave, and a particle at the lowest part of its vibration is in the trough of the wave. These particles will again assume these positions when another wave replaces the first wave, for the particles themselves are not carried along by the waves; they rise and fall like boats as the waves of the sea pass under them.

The distance that the waves travel in one period is called a wave-length. If v denotes the velocity of the propagation of a sound-wave, λ the wave-length, and T the period, we have for uniform motion $\lambda = v\,T$.

If we express T as $\frac{1}{n}$ of the unit of time, we have $v = n\lambda$, or the distance traveled by the sound-waves in a unit of time is n wave-lengths. The general remarks that we have made in respect to wave-motion apply whether the motion of the particle is transverse to the direction of the wave or in the direction of the wave. Sound-waves consist of longitudinal vibrations, while light is produced by transverse vibrations. The propagation of a sound-wave can be illustrated in the following manner:

EXPERIMENT 157.—Wind a brass wire around a wooden cylinder of about one centimetre diameter, making the spires very near to each other. In this way make a spiral spring of about three metres long.

Attach one end of this spiral spring to a thin, hollow box which is fixed to a table; hold the other end of the spiral in the hand and force the spirals together through a short space near the hand. A wave made of spirals compressed together, and of spirals stretched apart, will travel along the bar and will be reflected from the box back to the hand. The compression of the spirals can represent the condensation, and the extension the rarefaction, of the air, when a sound-wave travels through air. In the brass wire the elasticity of the metal aids the propagation of the wave. In air the elasticity of the air performs the same function.

EXPERIMENT 158.—Hang up ten or twelve magnetized bars of steel, each by a bifilar suspension, in line, with their opposite poles very near each other but not touching each other. Set the end-magnet to swinging in the direction of its length, and its vibration will be transmitted from magnetic particle to magnetic particle. In this experiment the magnetism of each pole represents the elasticity of the air.

Thus, in investigating wave-motion, as it is illustrated by the phenomena of sound, we need to fix our ideas by the ideas of time and space and velocity. To a person standing above a tuning-fork, who could also see the to-and-fro excursions of the fork, the particles would move along a line the length of which gradually decreases as the sound dies away. The same person lying on his back, and looking up at a pendulum which is also executing its swing to and fro, would see the pendulum-bob move to and fro along a line which is also continually decreasing in length. Let one also look edge-

wise at a point on a wheel which is revolving uniformly: this point will move up and down a line which is the diameter of the wheel. In this case the amplitude of motion of the point is the radius. We can fix the position of this point on the circumference of the wheel at any time by knowing its angular distance from the center of the circle, estimating the angle from any fixed line or diameter of the circle we may choose. The time that it takes the point to travel around the circle from one point back to this point again, passing through 360°, we have seen, is called the period.

EXPERIMENT 159.—Cut a rectangular slit in a piece of card-board; move a pencil to and fro in this slit in time with any pendulum, and at the same time draw a sheet of paper beneath the pencil: we shall obtain a sinuous curve.

Now we have seen in the case of pendulums that the time of swing is independent of the amplitude; that is, a pendulum executes its swings, if they are not very large, in the same time. The elasticity of the prong of a tuning-fork serves to exhibit pendulum-movements. We know that, under certain limits, an elastic body when distorted returns to its original position; moreover, that the force of restitution, or the force necessary to bring the body back to its original position after a force has distorted it, is directly proportional to the amount of distortion. When a tuning-fork or any rod is vibrating, the forces of elasticity urge the prongs toward the position of equilibrium. They vanish when the prong passes through this position of equilibrium, and they increase gradually as the prong advances on the opposite side of the position of equilibrium.

If we consider the manner of propagation of waves, we perceive that one wave can interfere with another. Let two waves of the same length proceed in the same direction; if they coincide in their phases, they strengthen each other, for their crests and troughs would then coincide. If, however, one wave has a phase one half a wave-length behind the other, the waves neutralize each other and no sound is produced.

EXPERIMENT 160.—Arrange glass or smooth tin tubes, as shown in Fig. 124, connecting the T-joints

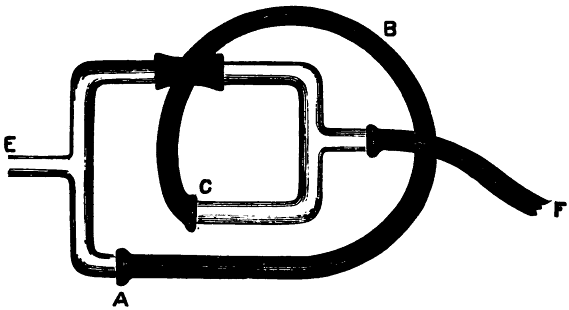

FIG. 124.

by rubber tubing. On sounding a tuning-fork at *F*, no sound will be heard at *E*, if the length *A B C* is half the wave-length produced by the fork.

If the length *A B C* could be easily varied, and if there were no errors due to irregularities and friction of air in the tubes, this method would enable us to measure the wave-length of sound. Compare the wave-length obtained in this way with that obtained in Experiment 154.

If two waves proceed in the same direction, the waves differing in wave-length, it is evident that

at some part of their path the crests of the two waves may partially coincide and at another part the phases may differ by a fraction of a wave-length. Thus the two waves might coincide at *A* (Fig. 125) and interfere at *B C*, just as two men walking beside each other with steps of unequal length, and

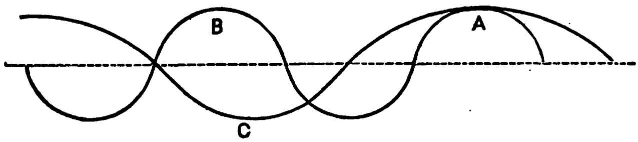

FIG. 125.

not endeavoring to keep step, will fall into step and out of step at regular intervals. The ear, listening to two sources of sound giving forth waves of different lengths, will detect what are called beats, which are produced by these coincidences and interferences.

EXPERIMENT 161.—Support two equal tuning-forks, which give about 250 vibrations per second, over two vertical glass vessels. Sound the tuning-forks and gradually fill the glass vessels with water. The forks being equal, they will both be re-enforced when the water stands at the same level in the two equal vessels. Now load the prongs of one of the forks with a bit of wax. The column of water which will respond to the loaded fork must now be changed. When it re-enforces the loaded fork, beats will be heard, and a condensation from one fork will reach the ear at the same time that a rarefaction comes from the other. If one fork should vibrate 100 times a second, and the other 101, there would be one beat a second. The number of beats

is equal to the difference in the number of vibrations of the forks per second.

EXPERIMENT 162.—Velocity of sound in air or in a gas. A tall glass cylindrical vessel is filled with gas by displacement. Carbonic-acid gas can be easily used, or even common street gas. While the gas is flowing into the cylinder gently through a rubber tube attached to a receptacle of the gas, sound a tuning-fork over the mouth of the vessel, and pour in water until the note of the fork is greatly strengthened. The length of the column of gas in the tube, multiplied by four, gives the theoretical length of the sound-waves of the note of the fork, in the gas which is employed. It is found necessary to add to this one fourth of the diameter of the tube, or $v = 4\,n\left(l + \frac{d}{4}\right)$, in which d is diameter of the tube, l length of column of gas, n rate of vibration of fork. A tuning-fork giving from 250 to 300 vibrations per second can be employed. Take the revolving wheels we have used in Experiment 49 and replace the more rapidly revolving one by a thin disk of sheet-iron or card-board, and punch holes about five millimetres in diameter in this disk around concentric circles. Arrange a glass tube, one end of which is connected with a rubber tube, opposite any row of holes, and blow through the rubber tube while the wheel is rapidly rotated. The pulses of air thus produced will give a note which rises in pitch as we increase the velocity of revolution of the perforated wheel. Place opposite to the row of holes through which we blow, a glass tube open at one end and closed at the other by a cork. Place this cork so that the column of air in the tube shall re-enforce

the sound given by the pulses of air. We can maintain the revolution of the perforated wheel fairly uniform by maintaining the loudest effect of the re-enforcement. In this way we can compare the sound of a tuning-fork with that of this apparatus, which is called a siren. Knowing the number of holes through which we blow, and ascertaining the rate of the wheel which we turn, we can compute the number of pulses of air which are sent into the resonating tube in a second. This will give the number of vibrations of the tuning-fork which produces the same note. It will be noticed that the effect on the air is the same as if a vibrating body was setting it in motion. The pulse of air driven through an opening causes a condensation, and the elasticity of the air in turn produces a rarefaction.

A description of a more complicated instrument called a siren can be found in any work on natural philosophy. The instrument we have described embodies all the principles of a siren.

It has been proved that $v = \sqrt{\frac{E}{D}}$, v being the velocity of sound, E the elasticity of the medium, and D its density. This formula is sensibly true for substances which radiate heat readily, but air and most gases are very poor radiators; and Laplace showed that the value for v must be multiplied by a factor which is the ratio of the specific heat at constant pressure to that at constant volume. The compression of the atmosphere in front of a sound-wave increases its temperature, while the rarefaction or extension behind it lowers the temperature. This increase of temperature increases

the elasticity in front, and the diminution of temperature diminishes the elasticity of the air behind, the wave-surface. The velocity depends upon this difference of stress, and is consequently greater than if the heat were quickly radiated. The ratio of the two specific heats is 1.41. Hence the velocity of sound in a gas is $v = \sqrt{\frac{E}{D} 1.41}$.

EXPERIMENT 163.—The change of temperature due to compression of air and its rarefaction can be shown by the following apparatus: A tube, of smooth internal bore, is bolted air-tight upon a support (Fig. 126), a leather washer being placed beneath the flange of the tube and the support. Pass through the support two wires leading from a thermo-electric junction; the other junction is immersed in water of a constant temperature outside. Connect these junctions with a low-resistance astatic galvanometer, *G*. On quickly compressing the air in the tube, by means of the air-tight piston, it will be seen that the air within the tube is heated, and on lifting the piston it is cooled.

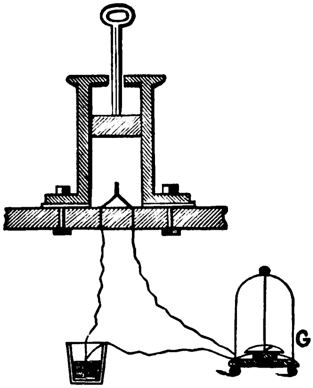

FIG. 126.

In this experiment, suppose that we have a tuning-fork instead of the piston, and a resonating tube; it is evident that we must have heat attending the compression produced in front of the sound-wave,

and loss of heat accompanying the rarefaction behind the wave.

The apparatus of Experiment 163 suggests how the mechanical equivalent of heat might be obtained by its use. Suppose the apparatus, before it is bolted to a support, should be placed in water, and the piston should be moved rapidly backward and forward one hundred strokes, and the increase of temperature of the water produced by the friction of the piston against the sides of the tube be measured. Then replace the apparatus upon its support, and measure, by means of the thermo-electric junctions, the increase in temperature after one hundred strokes. After subtracting the amount of heat due to friction, we have the heat due to the work done in compressing the air. The mechanical equivalent of heat has been obtained in this way. If we could make a tuning-fork vibrate in a closed tube and could measure the increased temperature of the air, and also the work done on the tuning-fork and its increased temperature, we could also measure the mechanical equivalent of heat.

Sound is transmitted through metals much more rapidly than through air or gases. The wire or string telephone is a familiar illustration of this. Connect the centers of the bottom of two tin cups by means of an iron wire thirty to a hundred feet long, supporting the wire if necessary at one or two points by fine thread. On speaking into one cup, a listener at the other can hear even a whisper. The interval between sounds spoken at one end and heard at the other, both through the iron wire and the air, is easily distinguishable when the distance is considerable.

We are familiar with the difference between elastic and non-elastic substances. The pendulum-movements of the particles of an iron wire must be very rapid; and, moreover, if the wire is very elastic, the particles in executing this movement must come to rest in their original position. The disturbance caused by a wave of sound in metals is very small, usually far within the limit of any permanent distortion of the metal. By a suitable weight attached to a wire, we can easily stretch an iron wire so that it will not contract to its original length. It will then be found to transmit sound less quickly than before: its elasticity has been altered. The elasticity is therefore connected with the relative position of the particles of the metal. The time of swing of these little particles considered as pendulums has been lengthened. In order to estimate elasticity, therefore, our experiments must be made on the metal within the limits of its deformation. In other words, the particles must be able, after swinging or being pulled out of their position of rest, to return to this position when the motion imparted to them has ceased. It is customary, therefore, to measure elasticity by the weight in kilogrammes which is necessary to hang on a wire of one square millimetre section in order to double its length. This weight is called the modulus of elasticity.

EXPERIMENT 164.—Another method of showing the propagation of longitudinal vibrations is that devised by Kundt (Fig. 127). This consists of a glass tube five or six feet long and one and a half inch in diameter, and another tube of glass of the same length, but one half an inch in diameter. One

end of the smaller tube is closed with a piece of cork just large enough to slip along the wider tube; the smaller tube also passes through a rubber cork which closes one end of the larger tube; the other

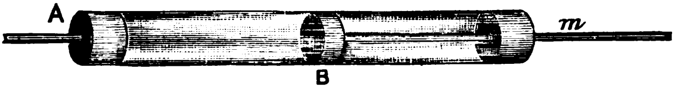

FIG. 127.

end of the large tube is closed by another rubber cork, the position of which can be regulated. The smaller glass tube is then pushed exactly half its length into the larger tube, and clamped at its middle point. A small quantity of lycopodium powder is sprinkled in the free portion, *A B*, of the tube. On exciting the free end, *m*, of the smaller glass tube by means of a dampened or resined cloth, it gives a note corresponding to its length; the lycopodium collects in little heaps which are separated by spaces of glass from which the powder is thrown; the heaps represent a node where there is no vibration. The rubber cork *A* is moved until no dust collects at its position. The column, *A B*, of air is then measured, and the number of spaces between the nodes counted. We then compare the note given by the small glass tube with that of a pitch-pipe or a siren. We shall then have, if $L =$ length of the column of air, n the number of spaces between the nodes, n_1 the vibration note of the glass, the wave-length in air $= \frac{2L}{n}$, and the velocity $=$ wave-length $\times$ number of vibrations $= \frac{2 L n^1}{n}$.

Care must be taken in this experiment to stroke

the smaller glass tube exactly in the direction of the axis of the larger tube, otherwise a pressure will be brought upon the end of the larger tube which will break it. The same methods can be employed to obtain the velocity of sound in solids.

EXPERIMENT 165.—Instead of the glass rod of the previous experiment, a rod of the substance in which we wish to determine the velocity is substituted for the small glass rod. It is better, if possible, to make the new rod of the same length as the small glass rod. The new rod is excited in the same manner.

If we call l the length of the rod, l_1 the length of the vibrating column of air, n the number of spaces between the nodes, v the velocity of sound in air at the time of the experiment, v_1 the velocity in the rod, we shall have $v : v_1 = \frac{2\, l_1}{n} : 2\, l$, or, $v_1 = \frac{v\, l\, n}{l_1}$.

EXPERIMENT 166.—Another method of obtaining the velocity of sound by longitudinal vibrations consists in altering the length of a wire which is stretched between two points until the note excited by rubbing with a cloth covered with resin (or rubbing with the thumb and forefinger covered with powdered resin) is equal to that of a tuning-fork or of a given note of a pitch-pipe. The note excited should not be a high one, for it is difficult to distinguish the fundamental of a high note from its octave.

The method of longitudinal vibrations gives us also the means of obtaining the modulus of elasticity of a substance, for we have $v = \sqrt{\frac{E}{D}} = 2\ n\ L$. Hence, $E = 4\, n^2\, L^2\, D$. Since weight $W =$ volume

multiplied by density multiplied by g the attraction of gravitation, or, $W = VDg$, we see that $D = \frac{W}{vg}$; if, then, we call W the weight of a unit volume of the substance, we have $D = \frac{W}{g}$ (v being equal to unity). Hence $E = \frac{4n^2L^2W}{g}$. Since, in expressions for the modulus, the millimetre has been chosen as the unit of length, and the kilogramme as the unit of weight, it will be necessary to express W as the weight of a cubic millimetre in kilogrammes.

We have hitherto measured the velocity acquired by bodies falling under the influence of gravitation. The same methods as those already adopted can be used to determine the time between the passage of a body between any two points, moving with any measurable velocity. Thus, the velocity of a rifle-ball can be found by a suitable chronograph which registers the time of its leaving the rifle and the time it arrives at a given point. It can be made to break an electrical circuit at the moment of its departure, and make the circuit again on its arrival at a given place. The length of time it takes a wave of water to reach a given point, distant a measured interval from the point of disturbance, can be readily measured in the same manner. The interval which elapses between the actual discharge of the rifle and the sensation of its report can also be measured. This interval is evidently the time that it has taken the sound-wave to travel through the air.

EXPERIMENT 167.—Measure the velocity of the sound-wave in air. For this experiment the galvanometer used in Experiment 71 can be employed.

At a measured distance a sounding-board can be struck, the blow making a sound and breaking at the same time an electrical circuit connected with the galvanometer of the experiment. The moment the sound is heard at the instrument, the circuit can be made again, and the time between breaking and making the circuit can be estimated from the swing of the needle. The personal error of the observer must be estimated. This is the error that each individual makes in not closing the current on the instant of the arrival of the sound to his ears. It is called the personal error, and can be eliminated by receiving the sound from different distances. Another method of measuring the velocity of sound is as follows:

EXPERIMENT 168.—Hang up a pendulum of suitable length. Place a graduated arc behind it so that the pointer connected with the pendulum may pass over this arc. Station an observer near this pendulum. At a distance of several hundred feet place another observer, provided with a telescope, a hammer, and a board. At the instant this latter observer perceives through the telescope the pendulum swinging over the lowest point of the arc, let him strike the board with the hammer. When the observer near the pendulum hears the blow, the pendulum will be at some other point of its swing. Now let the first observer move toward or away from the pendulum until the sound of his blow comes to the observer, when the pendulum is again at the lowest point of its swing. It has then taken the sound just one swing of the pendulum to travel over the distance between the two observers. Knowing the time of the pendulum and this distance, the velocity of sound can be obtained from the ex-

pression $S = v\,t$, in which S is distance, v the velocity of sound, and t the time.

EXPERIMENT 169.—Obtain the velocity of sound in bars of wood, for instance, from longitudinal vibrations. Clamp a long bar of wood, of somewhat small section, in a horizontal position, by means of a vise, at its middle point. Attach a small, flexible stylus at right angles to its extremity. Adjust this stylus so that it just touches a plate of smoked glass. By its side also adjust the stylus connected with a fork of known number of vibrations. On stroking the other half of the wooden rod by a cloth or piece of leather covered with resin, longitudinal vibrations can be excited at the same time that the tuning-fork is sounded. On slipping the plate of smoked glass beneath the two vibrating bodies, and comparing their vibrations through the same spaces, we can obtain the number of vibrations of the wooden bar, and hence the velocity of sound in it.

In order to analyze the sounds of a tuning-fork, we have made it draw its vibrations on a rapidly-moving plate of smoked glass. By properly exciting the fork we can superpose upon the fundamental vibrations of the fork other vibrations, which are called harmonics. This is often accomplished as follows:

EXPERIMENT 170.—Procure a fork of considerable amplitude of vibration, so that its record is readily obtained. Excite it by drawing the bow across the prongs near their ends, and, while it is still sounding,

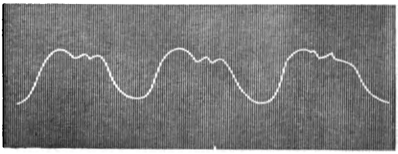

FIG. 128.

draw the bow also across the base of the V-shaped portion of the fork. The fundamental vibration will have harmonics superposed upon it, similar to those represented in Fig. 128.

Another way of showing the character of a sound-wave is by means of a rotating mirror; the method of using this can be illustrated by an apparatus which illustrates the character of the beat of the human pulse.

EXPERIMENT 171.—Stretch a string or wire tightly between two vertical supports. Twist a stiff light pointer around this string, so that one end of a little cross-bar at the shorter end of the pointer shall press somewhat tightly upon the pulse of the wrist; provide the other end of the pointer with a mirror; reflect a beam of light from the mirror to a little vertical mirror placed at the center of the whirling table used in the experiments on acceleration of gravity, and study the appearance of the sinuous curve in the revolving mirror.

Instead of studying the impulses or beats of the heart, suppose we should communicate the impulses of the air, produced by a source of sound, to a membrane whose movements to and fro, synchronous with the impulse of the air, should be made manifest by reflecting a beam of light from a little mirror placed upon the membrane to the same revolving mirror which we have used in the preceding experiment. This method might be used, but a better one is the following—a simple form of which is given in Mayer's "Sound," page 157:

EXPERIMENT 172.—"Take a piece of pine board, *A*, one inch (25 millimetres) thick, one and a half inch (38 millimetres) wide, and nine inches (22.8

centimetres) long. One inch from its top bore a shallow hole, one inch in diameter and one eighth of an inch deep. Bore a like shallow hole in the

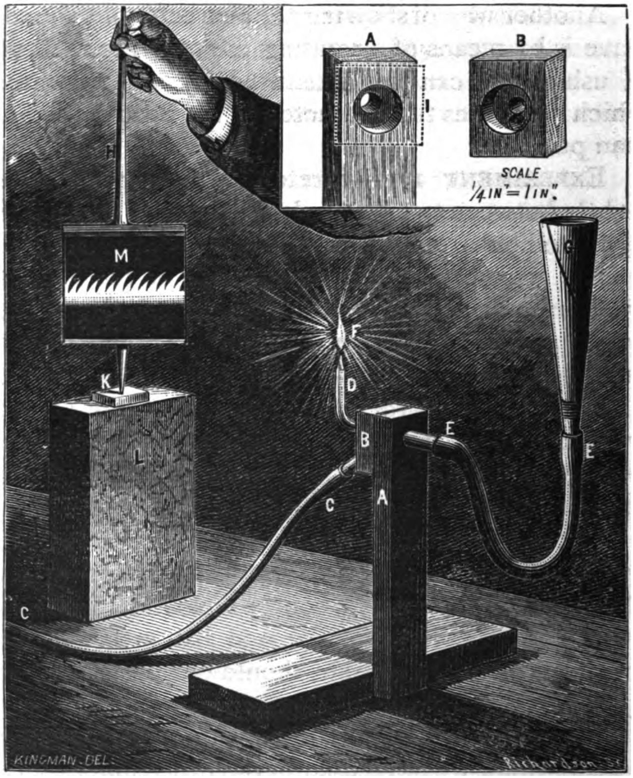

FIG. 129.

block *B*, which is three fourths of an inch thick, one and a half inch wide, and two inches (51 millimetres) long. Place a half-inch center-bit in the center of the shallow hole in *A*, and bore with

it a hole through the wood. Into this fit a glass or metal tube, as shown at *E*. Bore a three-sixteenth-inch (five millimetres) hole obliquely into the shallow hole in *B*, and into this fit the glass tube *C*. Then bore another three-sixteenth-inch hole directly into the shallow hole in *B*. Into this hole fit a glass tube, bent at right angles, and one end drawn out into a fine tube. Get a small piece of the thinnest sheet-rubber you can find, or a piece of thin linen paper, and having rubbed glue on the board *A*, around the shallow hole, stretch the thin rubber or paper over the hole, and glue it there. Then rub glue on the block *B*, and place the shallow hole in this block directly over the shallow hole in *A*, and fasten *B* to *A* by wrapping twine around these blocks. Thus you will have made a little box divided into two compartments by a partition of thin rubber. Fasten the rod *A* to the side of a small board, so that it may stand upright. Attach a piece of large-sized rubber tube to the glass tube *E*, and into the other end of the tube stick a cone, made by rolling up a piece of card-board, *G*, so as to form a cone eight inches long, and with a mouth two inches (51 millimetres) in diameter." On singing into this cone and revolving the mirror, the little flame will be seen to respond to the impulses upon the membrane of rubber, or the partition of paper, and the form of different impulses can be studied by observing the serrated band of light in the revolving mirror. With this apparatus study the sound of do, re, mi, fa, sol, la, si, do.

EXPERIMENT 173.—A suitable resonator can be made cheaply by making a tin tube, *T* (Fig. 130), slide into another, *T′*, and providing *T′* with a rub-

ber tube and ear-piece. The length of the column of air which re-enforces any note sounded over T can be read off on the side of the tube T. A series of these resonators, of different diameters, can then be used to analyze the harmonics or overtones of any source of sound.

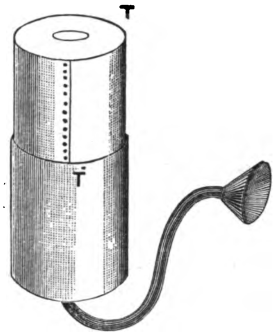

FIG. 130.

In our study of sound we have thus far treated it as simply a question of velocities of propagation of wave-motion, and we have employed the simple formulas of uniform motion, the principal one of which is space = velocity × time, or $S = vt$. In mechanics, after treating the subject of velocity in reference to the space passed through, we dealt with the problems of the combination of several velocities, either acting in the same line or in lines inclined at different angles. In the study of wave-motion of sound we have also to consider the composition of velocities.

The motion of a pendulum can be likened to the motion of a nail in the rim of a wheel which is spun with uniform velocity upon a fixed axle. The swing of the pendulum would then correspond to the diameter of the wheel. In the case of the wheel we can estimate the position of the nail at any time by its angular position with reference to the center of the wheel: this we call its phase. If we had two wheels with two nails, one in each rim, we could estimate their position relative to each other by

measuring their angular distance from the horizontal diameter of the wheels. If these angular distances are equal and of the same sign, the nails would be in the same phase.

If we could see the motions of the prongs of a tuning-fork, looking vertically down upon its ends, we should see a bright bead upon the fork, moving in the same way as the nail upon our wheel, and by comparing this motion with that of a point on a circle, of which the diameter represents the swing of the tuning-fork, we could compare the periods of two tuning-forks.

Instead of lying on our back and watching the movement of a pendulum, we can make a pendulum draw its own movement; and, by combining two pendulums, we can obtain their combined movement.

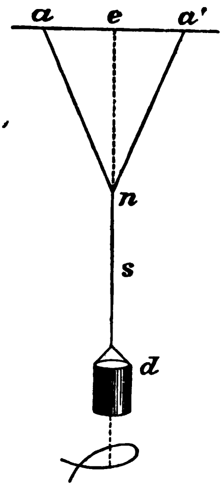

FIG. 131.

EXPERIMENT 174.—To the middle of the loop formed by a string (Fig. 131), the ends of which, *a a'*, are attached at some distance apart, attach another string, *s*. At the extremity of this latter string tie a tin cup provided with a small pin-hole in its bottom. This apparatus can be made to move so as to describe two lines at right angles with each other, which represent the amplitude of the pendulum whose length is *d e*, and the pendulum whose length is *n d*. If the vessel at *d* is drawn a little out of the direction of the swing

of the pendulum *d e*, the sand will describe figures which represent the composition of the two pendulum movements.

It is evident that, if we could combine the movements of two tuning-forks in this manner, we could obtain similar figures. Before showing how this can be done, let us turn to the methods of mechanics and devise a machine by which we can draw the curves which result from the combination of two uniform motions or pendulum vibrations at right angles with each other.

EXPERIMENT 175.—Upon a board (Fig. 132), provided with supports, so that it can be made to stand vertically, fix two wheels, *A*, *B*, upon axles. Connect these wheels by a belt, so that the movement of one is accompanied by a movement of the other. To these wheels attach two crank-arms or piston-rods. The rod attached to the wheel *B* moves a stage, *s*, upon which a plate of smoked glass, *g*, is mounted vertically, to and fro through a suitable amplitude. The rod attached to *A*, in the same way, moves a slide, *o*, in guides, $n\,n^1$, to and fro in a direction at

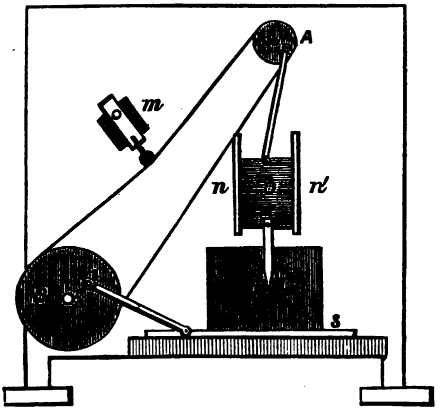

FIG. 132.

right angles with the movement of the rod connected with *B*. A stylus attached to the slide *o* will then draw on the smoked glass the figures resulting from the different rates of movements of the wheels *A* and *B*. The amplitudes can be changed by altering the point of attachment of the piston-rods, and different wheels can be slipped upon the two axles. A little wheel, connected with a slide at *m*, can be clamped in different positions, and serves to tighten the belt.

EXPERIMENT 176.—A small lens, about one centimetre in diameter, is fixed by bees-wax to the prong of one tuning-fork *F* (Fig. 133), which is made to vibrate in a vertical plane. The prong of another tuning-fork, *F′*, which is made to vibrate in a horizontal plane, is blackened, and a small bright dot is made on the blackened surface by a scratch. On looking through the lens, a curve will be seen, which is due to the composition of the vibrations of the two forks. Load the prongs of one fork by sticking small weights to them until the curve is a figure of 8. In this case, one fork gives the octave of the other. Prove this by comparing the vibrations of the fork by Experiment 154.

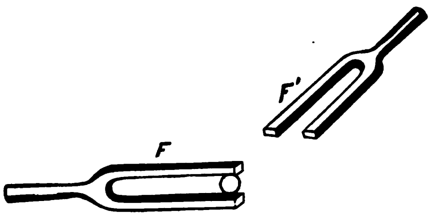

FIG. 133.

Our study of sound-waves, therefore, resembles our study of velocity and of motion. We have been concerned principally with the question, How many vibrations a second does a sounding body make?

We have found it necessary to use certain terms, such as pitch, amplitude, phase, and wave-length, in order to fix our ideas, and to represent what takes place when waves interfere with each other. The graphical methods which we have used illustrate the combination of periodic motion, and the sounding-flames illustrate the fact that every wave of sound consists of a fundamental wave upon which are superposed smaller waves, just as a long wave on the surface of water can carry smaller waves or crispations with it. The discussion of the character and influence of these superposed waves upon the nature of the sound given forth by a vibrating body would carry us into the subject of music, and we have, therefore, merely indicated the method of study which would serve to give us the relative value of sounds.

Since we are studying motion and energy, the question now arises, How can we obtain the energy of a vibrating body? This is the question we asked ourselves as soon as we were able to measure the velocity of a body acted upon by a force.

If we strike together two pebbles in air, waves of sound spread out from the source of sound in all directions, forming spheres, the surfaces of which pass through the alternate condensation and rarefaction of the air. These spheres are continually enlarging. Each wave carries with it its original amount of energy, and since this vibratory energy is shot forth, so to speak, in radiating lines, a cone of radiating lines would be cut by the continually enlarging spheres, and the same amount of energy would be carried through the successive intersections of the surfaces of the spheres and the cone.

These surfaces are inversely as the squares of the radii of the spheres. The intensity of sound propagated through equal areas on any two spheres would therefore be inversely as the squares of the distances of those equal areas from the source of sound.

EXPERIMENT 177.—Arrange a microphone by gluing a piece of smooth carbon to the center of a thin circular disk of paper or sheet-metal (Fig. 134), and, suspending against this another piece of car-

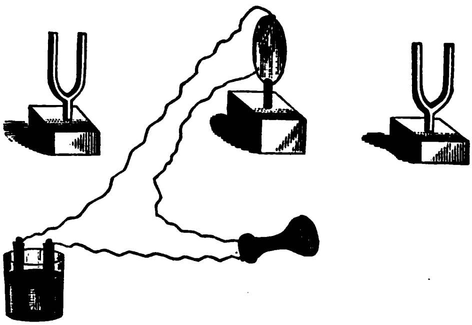

FIG. 134.

bon, connect these two pieces of carbon with a telephone and a battery. Move the two sources of sound until no sound is heard in the telephone: the intensities of the source of sound will then be equal at this distance of the microphone. We can thus compare one source of sound with another whose intensity is known.

A similar method to the above is used to compare the intensity of two lights.

Although it is comparatively easy to compare the relative intensity of two sounds, it is difficult to

obtain an absolute value of the amount of energy due to vibration. The work done by pendulums moving under the influence of gravitation can be readily estimated. Most vibrating bodies, however, are complex in their structure. The work done may be called internal work, and we can not measure it so simply as we have done in the case of the simple pendulum.

If we take any particle of a string which is vibrating transversely, we see that, as it swings to and fro across a horizontal line, on one side this line it is gaining kinetic energy and losing potential energy, and on the other side of the horizontal line this operation is reversed. The analogy between the vibrations of a particle of the string and that of a pendulum is complete. There are, however, various considerations of elasticity of the vibrating body which make the calculation of the transformation of energy in the case of vibrating bodies, like this string or the prongs of a tuning-fork, somewhat difficult. These calculations would lead us into the subject of molecular physics. We soon perceive a difference in the transmission of energy by means of vibrations. We can apply the force directly to keep up the energy of vibration, or we can set a body in vibration by transmitting the energy through the intervening media.

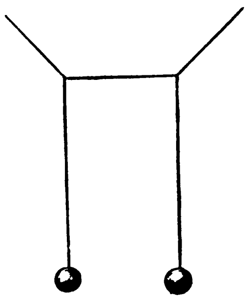

FIG. 135.

EXPERIMENT 178.—Hang up two equal balls (Fig. 135), by strings of the same length, from the same

string which is stretched between two points of support; hold one ball, and set the other to swinging; then remove your hand from the first ball: it will be noticed that in a short time the second ball will give up its energy to the first ball and come to rest. In turn the first ball will communicate its energy to the second and come to rest, and this operation will be repeated until the friction and air resistance will consume all the energy that has been communicated to the system.

EXPERIMENT 179.—Hang up two long magnets by equal bifilar suspensions, so that their opposite poles may be very near but not touching each other. Set one magnet to swinging, and hold the other for a moment: we shall perceive here, again, a transformation of energy from one magnet to the other by sympathetic vibration, principally through the medium which transmits magnetic pulls and pushes.

CHAPTER XIV.

LIGHT.

THE phenomena of heat and light are so closely allied that it is impossible to regard them as separate manifestations of energy. A heated substance gradually increases in luminosity as more work is done upon it, and finally emits a bright light. We can measure the heat produced in terms of the work that is done; there is an exact equivalence between the two. We seem, therefore, to have the light without the expenditure of any external work. The light of a candle can be seen several miles, but its heat can be measured only within a few feet of the flame. By passing an electrical current through a fine platinum wire or filament of carbon, we can produce a light equivalent to twenty candles. We can measure the heat equivalent of the current, and we find that it is equivalent to the work done in producing the current. In this case also we seem to have the light without the production of work. This is merely because our means of measuring the energy in the shape of light are not delicate enough to measure it in comparison with the enormous amount of energy sensible in a body raised to the temperature of red heat.

The common experiment of bringing the sun's rays to a focus by means of a lens, and burning

paper at this focus, shows us that light and heat invariably accompany each other in the case of the energy transmitted to us from the sun. The simplest mechanical analogy that occurs to us is that of the blow of a six-hundred-pound shot against a ballistic pendulum, accompanied by the blow of a small rifle-ball, both coming with the same velocity. It is evident that the ballistic pendulum which would measure the energy developed by the blow of the six-hundred-pound shot would not be sensitive enough to measure that of the accompanying rifle-ball, although the effects produced by the rifle-ball might be perfectly sensible in other ways than in producing a swing of the large mass of the pendulum. In the same way a large wave in the ocean may bear upon its surface many smaller waves which move with it and with the same velocity. The large wave can lift a steamship of many tons weight; while the smaller waves, producing no appreciable effect upon the steamship, are yet doing work in lifting the particles of the water.

The conception that the sun's energy comes to us in the form of waves seems to be the only admissible one. The conception of wave-motion requires also that of some medium in which this wave-motion can take place. Thus we are led to the hypothesis of an ether which fills all space, and is found even between the particles of solid bodies. The energy of the sun is transmitted to us by wave-motion, the large waves giving more of the sensation of heat than of light, while the shorter waves give more the sensation of light than of heat, both the sensations of light and heat depending upon the length of waves which are propagated in the ether, accept-

ing the hypothesis of an ether and the establishment of waves in it by the sun.

In our study of light we are necessarily occupied at first in tracing the path of the rays of light from various objects to our eyes, and also in observing the effect of the bending of these rays of light by different media. Thus, our study at first is a study of diagrams or configurations, and is necessarily geometrical. It is analogous to our study of diagrams of velocities, and at first does not introduce the idea of energy. Since we use lenses and prisms to concentrate or to break up beams of light in order to study the distribution of energy in a beam of light, it is necessary to have clear ideas in regard to the action of lenses and prisms.

In the first place, we notice that rays of light travel in straight lines; that a smooth surface, like a mirror or unruffled water, reflects the ray that impinges upon the surface, just as a hard wall reflects an elastic ball that strikes it at an angle. If the surface is roughened, each little facet of the surface diffuses the light by reflection, and only certain particles reflect rays to the eye. If all surfaces were brilliantly polished, we should not be able to see the form of objects distinctly, for we should notice merely lines of great brilliancy such as are seen on a polished metallic cylinder in the sunlight. Since polished surfaces reflect light in different directions, a person on one mountain can converse with another person on a distant peak by flashing the image of the sun, by means of a mirror, to the eye of the distant observer, having previously arranged an alphabet of short and long duration of flashes, similar to the Morse telegraphic alphabet. A piece of plate-

glass will give one reflection from its first surface and several from its second surface. One ray, becoming imprisoned, so to speak, between the two surfaces of the glass, bounds backward and forward, giving what is called multiple reflections. A similar case of internal reflections in a tube is afforded by the chimney of a student-lamp. Concentric rings of light and shade will be seen above the chimney on the wall. Holding a candle in front of a mirror and looking at its reflection at a considerable angle of reflection, several images of the candle will be seen; these images are due to the reflections from the inner surface of the glass.

EXPERIMENT 180.—To prove that the angle of incidence is equal to the angle of reflection, place a piece of looking-glass in a vertical position at the center of a large graduated circle (a paper or horn protractor, such as surveyors use, can be employed), or a circle divided into equal parts by a pair of compasses. Attach a long pointer to the little mirror, and observe that, when the mirror is turned through a certain angle, a beam of reflected light moves through twice this angle.

EXPERIMENT 181.—Place a screen, *B* (Fig. 136), in front of a lamp, with a little pin-hole in the blackened lamp-chimney. Move a lens (a spectacle-lens will answer) between the lamp and the screen, *B*, until an image of this illuminated pin-hole is seen sharply defined on the screen. Then place a little mirror, *a m*, between the lamp and *B*, and move the lamp along a line perpendicular to the direction of the mirror, making the distance $CE = DE$. A sharp image will still be seen on the screen *B*. This shows that light reflected from a mirror

appears to come from a source as far behind the mirror as the real source is in front.

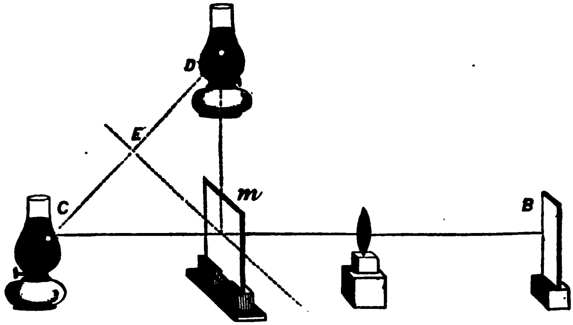

FIG. 136.

EXPERIMENT 182.—Compare the areas of the three screens *A*, *B*, and *C*, which in turn intercept all the light that comes from a small opening in a screen placed in front of a lamp, and also see how these areas are related to their distances from the hole *D* in the screen. Let *A* be one foot from the screen, *B* two feet, and *C* three feet. The screen at the distance of one foot intercepts all the light that falls upon it.

If we call the whole amount of light that is distributed over the area of any one circle *M*, we shall have in each square centimetre of *A*, $\frac{M}{\pi r^2} = I$, and on *B*, $\frac{M}{\pi r_1^2} = I_1$, since the same amount of light is received by both circles, but less per unit of area in the two cases.

Hence $\frac{r_1^2}{r^2} = \frac{I}{I_1}$. We can also prove by measurement that $\frac{r_1^2}{r^2} = \frac{d_1^2}{d^2}$, or $\frac{I}{I_1} = \frac{d_1^2}{d^2}$.

If, now, we could measure the quantity M in absolute measure, we could obtain an absolute standard for the estimation of the intensities of different lights. Unfortunately, there is great difficulty in obtaining such an absolute standard. It is, however, very easy to measure the relative intensities of different lights.

EXPERIMENT 183.—Place in front and quite near a vertical screen made of tissue-paper or of porcelain, a vertical rod. Place the two lights to be compared on each side of the perpendicular to the screen passing through the rod, so that the horizontal lines drawn through them and the rod will form equal angles with this perpendicular, and throw shadows of equal breadth upon the screen. Move one light away from or toward the screen, until the shadows are of the same intensity. The intensity of one light is $\frac{M}{d^2}$, and that of the other $\frac{M_1}{d_1^2}$. These intensities are made equal by the experiment; hence $\frac{M}{d^2} = \frac{M_1}{d_1^2}$, or $\frac{M}{M_1} = \frac{d^2}{d_1^2}$. This method is due to Rumford.

EXPERIMENT 184.—Cover a circular disk of paper, except a small circle about two centimetres in diameter in the center, with a solution of wax in benzine. This will render the paper transparent. Mount this disk so that it can slide horizontally along a graduated bar; place a standard candle—a sperm-candle weighing one sixth of a pound and burning one quarter of an ounce per hour—at one end of this bar, and the light to be compared with it at the other end. Slide the disk along the bar until the disk is equally illuminated on both sides.

When this is done, the white-paper spot at the center of this disk will no longer be seen. In this case we have made the intensities of the two lights equal, and here we also have $\frac{M}{M_1} = \frac{d^2}{d_1^2}$. This method is due to Bunsen.

It will be observed that the methods thus far given for obtaining the intensities of two lights are simply relative. There is no absolute standard of light. The lights produced by different materials are all different in tint, and it is found very difficult in practice to compare the intensities of light of different colors. The best method of connecting our measurements of the intensity of light with our measurements of energy in general would seem to be by comparing all lights with an electric incandescent light produced by passing an electrical current of known strength through a known resistance. In this case we should know how much energy is necessary to produce the standard light, and other lights could then be compared with such a standard. Even in this case, however, we do not know the mechanical equivalent of the light-waves. Moreover, we should still be confronted with the difficulty of comparing lights of different colors. It is also difficult to maintain a constant light by an electrical current. The molecular work done by light of different wave-lengths in photography might give us a standard if we were able to estimate this work in absolute units. If we had also a medium, in passing through which a definite amount of light would be absorbed, and if we could estimate the amount of internal work done by the waves of energy in passing through such a medium, we

could use this medium as a means of estimating the strengths of lights. Our means of measuring the intensities of light by absorption are only relative. In order to measure the amount of light that is absorbed by a medium, this experimental determination having a certain analogy to the determination of the amount of energy consumed in the transmission of mechanical power, we can proceed as in the following experiment:

EXPERIMENT 185.—Two mirrors, *m m'* (Fig. 137), are placed upon a graduated bar. A greased disk can slide on this bar between the mirrors. A light, *L*, suitably placed behind the angle of a screen, can be reflected by the two mirrors upon this disk. This happens when the latter is at the middle of the graduated bar, the mirrors at an angle of 45° with the bar, and the distance from the disk to the light equal to its distance from each mirror. No spot can then be seen upon the disk. Interpose, however, the absorbing material at either side of the disk, as at *A*, and it will be found that the disk *g* must be moved nearer the mirror *m*. It is as if a light of feebler intensity, I_1, placed at a distance *g m L* along the graduated bar, is to be compared with a light of intensity *I* at a distance *g m' L* in the opposite direction (since to the eye placed at *g* the light at *L* appears

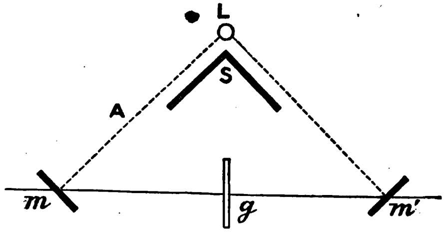

FIG. 137.

to come from behind the mirror *m* at a distance along the scale equal to mL. If we call the distance $m' L = d$, and the distance between the mirrors $=$ a, and supposing this latter distance to be divided into centimetres, the readings, n, running each way from the center of a, we shall have $\frac{I}{(d+a+n)^2} = \frac{I^1}{(d+a-n)^2}$, which will give the ratio of the intensities of the two lights; or, in other words, the proportion of the light L that is absorbed by passing through the given medium.

There are many analogies between sound and light, since the phenomena of both are due to wave-motion. We can also measure the relative intensity of two sounds, but we have no mechanical standard or absolute standard of sound intensity. The nicely-trained eye can estimate the equality of two lights of the same tint to a very close approximation. The well-trained ear can also decide whether two notes sounded on two instruments are the same. But there is no absolute standard for the estimation of the intensity of sound. Therefore, there has been during successive years a change in the pitch to which concert instruments are tuned, for, having no absolute standard for the estimation of the intensity of sound, our relative standards necessarily fluctuate. In the same way our eyes get accustomed to strong light or weak light, and, having no unchangeable standard, the scale by which we estimate the intensity of light is subject to a similar variation to that which our ears experience.

Assuming that the sensations of heat and light are produced by wave-motion, we can apply the

laws of mechanics to the question of the movement of those waves, and the law of the conservation of energy to the transformation of the energy of wave-motion into that of molecular vibration. The first question that meets us in the discussion of the propagation of energy from the sun, is the same as that in regard to motion in general. What is the velocity of its propagation? It is found by experiments on wave-motion in different media that the law $V = \sqrt{\frac{E}{D}}$, in which V = velocity, E = elasticity, D = density, is a true one. If we could find the elasticity and density of the ether, we could then calculate the velocity of light. This, however, seems to be impossible at present. We therefore have to resort to direct experiments to determine the velocity of light. The principal methods are those of Fizeau and Foucault, descriptions of which can be found in most works on natural philosophy. The best values of the velocity of light seem to be those of Cornu and Michelson, which are, respectively, 300,400,000 and 299,940,000 metres per second. The methods of the later observers depend upon the measurement of the small interval of time which elapses between the reflection of light from one station to another and its return to its starting-point. This gives the time that it has taken the light to pass over twice the distance between the two mirrors.

When the velocity of light is known in one medium, it can be easily obtained in another by what is called Huygens's construction for the wave-front.

When a wave passes from one medium to another, the points where each point of the front of the wave strikes the line of demarkation between the two me-

dia become the center of propagation of new waves. Let $A\ B\ A_1\ B_1$ be the path of a beam of light (Fig. 138), and suppose that $A\ D$ is the wave-front, which is supposed to be so small as to be approximately straight. Let $A\ A^1$ be the surface of separation between two media, and let the velocity of light in the upper medium be V, and in the lower V_1. If the lower medium is the denser, the light will travel slower in it, and will be refracted toward the perpendicular to line or surface of separation $A\ A^1$. The following illustration of Herschel connects this property of light with our ideas of work: Suppose that $A\ B$ or DB^1 represent the line of advance of a column of soldiers, and let $A\ D$ be the front rank. When the column reaches a plowed field, those first striking the field at A will move more slowly, with the same expenditure of energy, than those at D, for instance. In order to preserve the order of marching, the front rank must therefore take a line $A^1\ m$, and the new line of march must be inclined at a different angle to the line between the smooth field and the plowed one. If v is the velocity of the soldiers along $A^1\ D$, we shall have $A^1\ D = v\ t$;

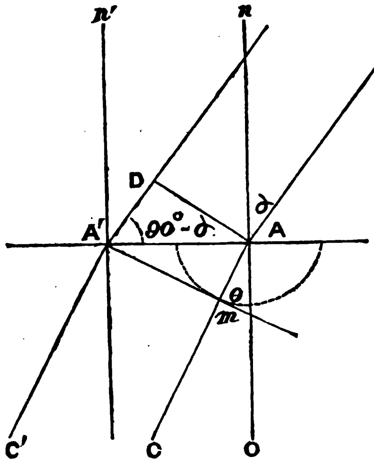

Fig. 138.

and if v_1 is the velocity of the soldiers in the plowed field, or along $A\ m$, we shall have $A\ m = v_1\ t$. Hence $\frac{A_1 D}{A\ m} = \frac{v\ t}{v_1 t}$, or $A\ m = \frac{v_1 A_1 D}{v}$; or, passing to the case of light, a wave starting from A will have advanced to m through a space $\frac{V_1 A_1 D}{v}$, while the original wave advances from D to A^1. If we now draw a circle from A with a radius $A\ m$, and then draw a line from A^1 tangent to this circle, we shall have the new wave-front, and the lines $A^1\ C^1\ A\ C$, perpendicular to this new front, will be the new direction of the ray. The angle which $A\ B$ or $A^1\ B^1$, the original ray, makes with the normals $A\ n$, $A_1\ n_1$ is α, or the angle of incidence. The angle of refraction, or $C\ A\ O = \theta$, is also equal to $M\ A_1\ A$, since their sides are perpendicular to each other. Hence we shall have, $A\ A^1 \cos(90° - \alpha) = A\ A^1 \sin \alpha = A^1 D = v\ t$; $A\ A^1 \sin\theta = A\ M = v_1\ t$. Hence, $\frac{\sin\alpha}{\sin\theta} = \frac{v}{v_1} = \mu$. The ratio between the velocities in the two media is called the index of refraction μ, and we see that this is the ratio between the sines of the angles of incidence and of refraction. Our determination, therefore, of indices of refraction is really a measurement of velocities. Our subsequent work upon the relations between heat and light requires the determination of this ratio of velocities.

EXPERIMENT 186.—Obtain a small cylindrical bottle of clear glass, a homœopathic vial for instance, holding about twenty centimetres, selecting one that is circular in section. Grind one side away and cement on a piece of plate-glass. Mount this vial at the center of a graduated paper circle, or

horn protractor (Fig. 139), and fill it with water. An illuminated pin-hole in a screen placed at *B*, will be seen by the eye at *A*. The angle which the incident ray makes with the perpendicular *Dc* to the surface of the liquid, is called the angle of incidence, and the angle which the refracted ray makes with this perpendicular the angle of refraction, and the ratio of the sines of these angles, or, in other words, the ratio of the perpendiculars *B D* and *A m*, is called the index of refraction, or $\mu = \frac{\sin i}{\sin r}$. Having obtained the value of μ for water, obtain it also for alcohol.

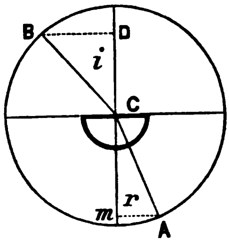

FIG. 139.

In order to experiment upon light, we must be able to make the rays of light parallel, after they have passed through a small slit, the image of which we use in studying the effects of refraction and dispersion. By reflecting sunlight into a dark room, through a pin-hole or narrow slit, we quickly perceive that the rays are no longer parallel after passing through the slit, but diverge, forming a cone or wedge of rays. It is evident that we can not experiment upon refraction with such a cone of rays. It is necessary to have a beam of which the rays are approximately parallel. We can accomplish this by placing the slit at the principal focus of the lens. For some distance the rays from an ordinary spectacle-lens will be approximately parallel, and if a prism be placed in the path of this beam, we can easily trace its passage

through the prism or refracting apparatus we employ.

In order to understand more refined apparatus, it is often useful to dissect, so to speak, such apparatus and form rough models of it. The following apparatus will give approximate results, and is useful to enable us to comprehend an instrument called a spectrometer, by means of which we finally measuse the length of the waves of light.

EXPERIMENT 187.—Having made the rays of a beam of light parallel by means of a narrow slit and a lens, as above described, place a glass prism or a

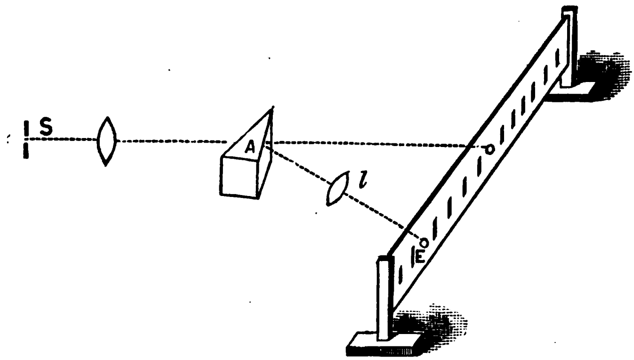

FIG. 140.

liquid prism in its path (Fig. 140), and receive the refracted beam upon a piece of oiled paper, or a ground-glass screen. The beam of light will be refracted to some point, *E*, which can be found by placing a lens, *l*, in the path of the refracted rays, thus bringing the rays proceeding from the slit, *S*, to a focus at *E*, and forming there an image of this slit. The tangent of the angle of deviation can then

be read in terms of the division of the proper scale, knowing the distance of the scale from the center of the prism. On turning the prism around a vertical axis passing through its center *A*, it will be found that the image *E* travels at first in the direction in which we turn the prism; but presently, while we continue to turn, the image ceases to turn in the same direction, and moves in the opposite direction.

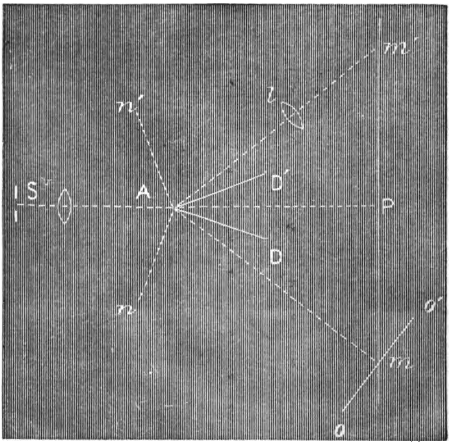

FIG. 141.

The position of the prism, when the refracted ray ceases to follow the motion of the prism around a vertical axis, is called the position of minimum deviation. When the prism is in this position the incident ray and the emergent ray make equal angles with the faces of the prism. For measurements the prism is usually placed in this position of minimum deviation.

It is necessary also to obtain the refracting angle

of a prism; this is the acute angle, away from which the light is refracted. In order to do this, place the prism so that its refracting edge shall be on the line passing through the slit (Fig. 141), which is also perpendicular to the screen. In this position, the beam of parallel light striking on the two edges of the prism which intersect at A, are reflected so as to make equal angles with the normals $A\ n$, and strike the screen at $m\ m'$, the positions of which can be found by moving a lens, l, until an image of the slit is seen at m and m'; $m\,A\,m'$ is then the tangent of twice the refracting angle A of the prism. $P\,A\,D' =$ $90° -$ angle of incidence, $m'\,A\,D' = 90° -$ angle of reflection; hence, $P\,A\,D' = m'\,A\,D'$. In the same way $m\,A\,D = P\,A\,D$, or $m'\,A\,D' + m\,A\,D = D\,A\,D'$, or $m'\,A\,m = 2\,D\,A\,D'$.

In case that the points m and m^1 are too far apart to be received on one screen, place a smaller screen, $o\,o^1$, at m or m^1, and drop a perpendicular from m to line $A\,P$; then $\frac{M\,P}{A\,P}$ will be the tangent of the angle of the prism, and the angle can be readily obtained from a table of natural tangents.

The index of refraction of a substance is generally determined by means of a prism. If the substance is a specimen of glass, it is made into a prism. If the substance is a liquid, it is poured into a hollow prism of glass.

When a beam of light is passed through a prism it is bent or refracted from its course. Let us examine the directions of the incident and emergent rays.

At the point B (Fig. 142), the angle of incidence of the ray $A\,B$ is i ($B\,n$ being the normal at B). The

angle of refraction is $D B c$ or r. The divergence of the ray $B D$ from $A B$ is $D B m$ or $i - r$, since the angle $m B c = A B n$. If we arrange the prism so that the emergent ray $D E$ makes the same angle

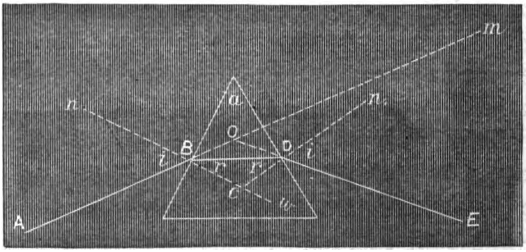

FIG. 142.

with the normal that $A B$ does, we shall have angle $c B D$ or $r = B D c$. Hence, $n_1 c w = r + r =$ angle a, since the normals are respectively perpendicular to the sides of the angle a, or $a = 2r$. In the same way from the triangle $O B D$ we have $i - r + i - r$, or $2i - 2r = E O m$, or δ; δ being the angle of deviation of the ray. Hence, $2i - a = \delta$, or $i = \frac{a + \delta}{2}$. But $\frac{\sin i}{\sin r} = \mu$, and $a = 2r$, or $r = \frac{a}{2}$. Hence,

$$\frac{\sin \frac{(a + \delta)}{2}}{\sin \frac{a}{2}} = \mu.$$

If we use a prism of very small angle, $\dfrac{\frac{a + \delta}{2}}{\frac{\delta}{2}} = \mu$, since the sine equals the arc in this case, or $\delta = (\mu - 1) a$.

A lens can be considered as made up of a number of prisms (Fig. 143); the rays of light coming from any source are bent by the prisms, and are

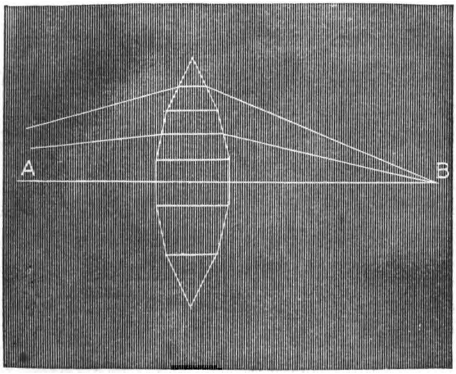

FIG. 143.

collected at a point which is called the focus. The principal axis of the lens is line *A B*.

The rays of light coming from the sun are parallel. The point on the principal axis of the lens where these parallel rays meet is called the principal focus.

EXPERIMENT 188.—Place a convex lens in the sunlight, and measure the distance of the principal focus from the center of the lens.

EXPERIMENT 189.—Substitute for the light of the sun a kerosene-light or a gas-flame, and receive the image on a ground-glass screen. It will be found that the distances of the images of the light vary when the lens is moved to or from the light. It will be found that $\frac{1}{F} = \frac{1}{d} + \frac{1}{d^1}$ expresses the law of the lens. Where F is the principal focus obtained by

preceding experiment, d and d^1 are the distances from the light to the lens and from the lens to the screen respectively.

This law of a lens can be obtained very simply by geometry. The propositions necessary to prove it are the following:

1. The exterior angle of a triangle is equal to the sum of the opposite interior angles.

2. Two angles are equal when their sides are mutually perpendicular.

3. The sine of an acute angle at the base of a right-angled triangle is equal to the altitude of the triangle divided by the hypothenuse.

4. The sine of a very small angle is equal to the arc which measures the angle.

Let $A B$ be a section of the lens (Fig. 144), $C D$ its principal axis, $C E F D$ a ray which passes from

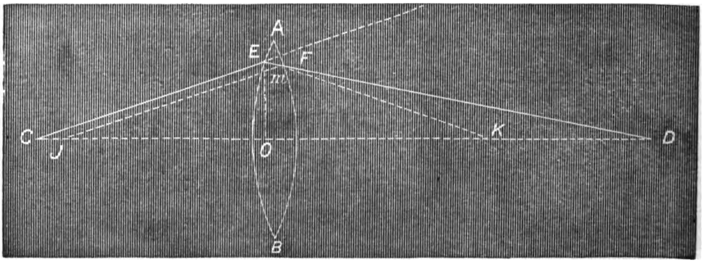

FIG. 144.

one conjugate focus, C, to the other at D, $J F$ and $K E$ the normals at F and E, $J m = r$, $K m = r^1$, $C E = p$, $D F = p^1$. Since $J F$ and $K E$ are perpendicular to the sides of the lens, we have $E m J = E A F = \theta$.

$$\delta = (\mu - 1)\,\theta, = (\mu - 1)\,(m J K + m K J).$$

Let fall a perpendicular from E upon $C D$; since

the angle θ of the lens is small, this perpendicular may be considered equal to and coincident with one let fall from F upon CD, or $\delta = (\mu - 1)\left(\frac{EO}{r} + \frac{EO}{r_1}\right)$, $\frac{EO}{r}$ and $\frac{EO}{r_1}$ being the sines of the angles mJK and mKJ, and the sines being taken equal to the angles for small angles.

We also have $\delta = ECO + FDO = \frac{EO}{p} + \frac{EO}{p_1}$ (the sines being taken equal to the angles)

$\therefore (\mu - 1)\left(\frac{1}{r} + \frac{1}{r_1}\right) = \frac{1}{p} + \frac{1}{p_1}$, eq. 1.

When one of the foci D is at an infinite distance, $\frac{1}{p_1} = o$, and we then have $\frac{1}{p} = \frac{1}{F} = (\mu - 1)$ $\left(\frac{1}{r} + \frac{1}{r_1}\right)$, eq. 2. From eq. 1 and 2 we see that $\frac{1}{F} = \frac{1}{p} + \frac{1}{p_1}$.

EXPERIMENT 190.—Place two convex lenses in front of a source of light, and let f be the focal length of one lens, A, and f_1 that of the other, B. Let d be the distance apart of the lenses A and B. Then, calling the distance from the lens B to the image D, we have $\frac{1}{D} = \frac{1}{f_1} + \frac{1}{f - a}$. If the lenses are close together, we have $d = o$, or $\frac{1}{D} = \frac{1}{f} + \frac{1}{f_1}$.

If the lenses have the same curvature, then $f = f^1$, or $\frac{1}{D} = \frac{2}{f}$.

If the second lens is a concave lens of focus f,

we then have $\frac{1}{D} = \frac{1}{f^1 - d} - \frac{1}{f}$, and if the lenses are in close proximity, $\frac{1}{D} = \frac{1}{f_1} - \frac{1}{f}$.

EXPERIMENT 191.—Determine the focal length of a concave lens by placing it in front of a convex lens of greater focus, and measuring the focal lengths of the combination. In this case the conjugate foci of the combination must be real.

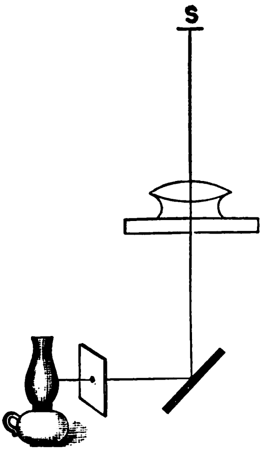

FIG. 145.

EXPERIMENT 192.—The index of refraction of a liquid can also be obtained in the following way: Place a concave lens of known radius of curvature upon a drop of the liquid on a plate of glass (Fig. 145). Reflect the light from an illuminated pin-hole, by means of an inclined mirror, through this combination of a plano-concave and convex lenses, to a screen, *S*, made of ground glass. The focal distance of the combination can thus be obtained; and we can obtain the focal length of the liquid lens from the formula $\frac{1}{D} = \frac{1}{f_1} - \frac{1}{f}$, since we have $\frac{1}{f_1} = (\mu - 1)\frac{1}{r}$ from eq. 2 by making r_1 the radius of the plane surface of one side of a convex lens equal to infinity, and hence $\frac{1}{r_1} = o$).

In order to ascertain the position and the size of

an image formed by a convex lens, it is only necessary to draw two rays of light from each extremity of the object, and find where these intersect. One of these rays is generally drawn through the center *O* of the lenses, and the other can be drawn to any part *C* of the lens; thus the image of *A* (Fig. 146) will be found at *A′*. In the same way *B′* can be found, and the size of *A′ B′* and its distance from *O* can be readily measured.

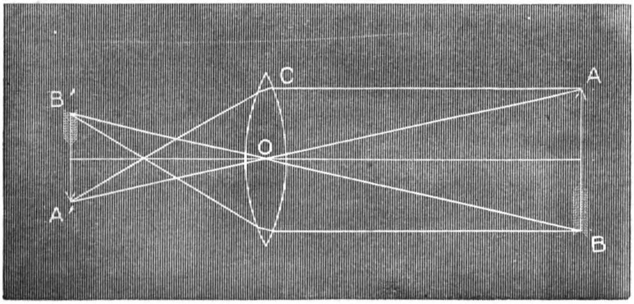

FIG. 146.

EXPERIMENT 193.—Ascertain the size of the images formed by a convex lens, with varying distances.

A bit of wire moved along the principal axis *A B* (Fig. 147) of a mirror will be brilliantly illuminated at the principal focus.

Prove that the lengths of image and object are directly as their distances from the mirror, and that their areas vary directly as the square of the distances.

The images produced by convex mirrors are not real images—that is, they can not be projected on the screen, and are only received by the eye.

EXPERIMENT 194.—Take a small Florence flask, or a small spherical bulb, and fill it with clean mer-

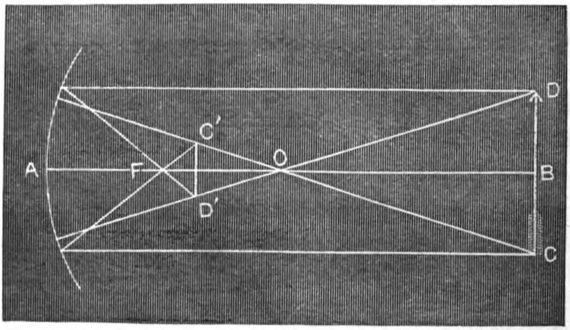

FIG. 147.

cury, and show that the distances of the virtual images and of the object from the center of curvature vary directly as their lengths and as the square root of their areas.

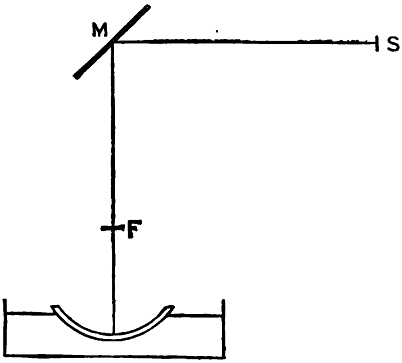

FIG. 148.

Let *O* be the center of curvature of a concave mirror of small angular aperture (Fig. 147), *A B* its principal axis, and *F* its principal focus. This focus can be found thus:

EXPERIMENT 195.—Select a small evaporating-dish which is approximately a portion of a spherical surface, and press it down upon the surface

of clean mercury, confining it by suitable pressure.

Incline a mirror at an angle of 45° (Fig. 148), so as to throw the image of a slit illuminated by the sun upon the concave mirror which is formed by the surface of mercury. A little alcohol should be first poured upon the mercury.

EXPERIMENT 196.—Make a concave lens, one side of which is plane—or, in other words, has a radius of infinite length—by pressing a glass evaporating-dish, whose surface is spherical, upon the surface of water which is contained in a vessel with a piece of thin plate-glass for a bottom. Keep the evaporating-dish in position by a suitable arrangement, and view some object beneath the lens thus formed. Prove that the formula in this case is $\frac{1}{F} = \frac{1}{p} - \frac{1}{p_1}$.

By means of the simple apparatus described in Experiment 187, we can determine the index of refraction of a prism. This simple apparatus is really a rude spectrometer, the slit *S* and the lens near it forming what is called a collimator—that is, an apparatus to make the rays of light parallel. It is usual to form this collimator by making a slit at one end of a long brass tube which is blackened inside. At the other end is placed the lens of long focus. The collimator is really a darkened room or gallery. The observing telescope is also a brass tube, one end of which is closed by a lens of moderate focus; and the other end is provided with an eye-piece suitable for observing the image of the slit formed by the lens *l*. The screen is replaced in the spectrometer by an observing tele-

scope, which moves around a graduated circle of brass.

Let us now return for a moment to the simple apparatus described in Experiment 187. Allow the refracted beam to fall upon a white screen. In this way we shall form a spectrum of the sun. This consists of a series of images of the slit in different colors. To form a conception of the effect of the prism in separating the beam of white light into light of different colors, let us suppose that a flock of birds, all advancing with the same velocity—red birds moving with long, flapping wings, and yellow birds with somewhat less powerful wings, and therefore flapping their wings oftener to keep up with the red birds, blue birds, green birds, violet birds, and so on—should fly toward us. All together, their colors, properly mingled as in the case of a beam of white light, would be undistinguishable. On meeting a dense, wedge-shaped medium, the more powerful birds would be less diverted from their path than the smaller birds, whose wings move much faster than those of the larger or red birds. In the effort to maintain their advance with the red birds, the birds whose wings move more rapidly are crowded away from the slower and more powerful birds and are dispersed, so that a gunner on the other side of this wedge-shaped medium would see the birds advancing in the order of the colors of the spectrum.

EXPERIMENT 197.—In order to project the solar spectrum upon a screen, deflect the sunlight into a darkened room through a narrow slit, about four centimetres long and two millimetres wide. Form an image of this slit on a screen by means of a lens of long focus.

Place a prism in the path of the convergent beams not far from the lens. On moving the screen, still keeping it at the same focal distance from the lens, the spectrum will be formed upon it, and, if the slit is of suitable width, the dark solar lines will be seen on the screen. The dark line (really two dark lines) in the yellow is called the sodium light, and is due to the absorption of the sunlight in passing through a layer of sodium vapor on the sun's surface. To illustrate this, and to make the line visible to a large audience, obtain some metallic sodium, and, having placed it in a small cavity in a piece of chalk, burn it in the flame of a Bunsen burner or spirit-lamp, which is placed directly in front of the slit, inside the darkened room. (The prism should be put in the position of minimum deviation.)

By means of a spectrometer the spectrum is seen to be crossed by a great number of fine, dark lines, which belong to the elements. These lines can be made bright lines by burning in front of the slit the different metals and cutting off the sunlight by a screen. The bright lines occupy the exact position of certain dark lines in the solar spectrum, and are reversed into dark lines in passing through their own vapor in the atmosphere of the sun. It is not possible to project on the screen, by means of the last experiment, the bright line of sodium, for the light is not strong enough. It is necessary for this purpose to burn the sodium in the electric light. In order that the bright line shall be reversed into a dark one, it is necessary that the sodium rays shall pass through the vapor of sodium, which is at a lower temperature than the source of the rays. A certain

portion of the energy of the waves of sodium light is, therefore, absorbed in striving to heat the sodium vapor which is at a lower temperature. The mechanism of this endeavor can be comprehended by a reference to pendulum movements. In order to increase the height to which a pendulum swings, it is evident that our blows must be timed so as to coincide in time with the swings of the pendulum. A heavy ball can be set in movement and maintained in vibration by attaching a fine thread to it and timing our impulses so that they coincide with the rate of swing of the pendulum. If, therefore, the molecules of sodium vapor, whose to-and-fro movements are like those of a pendulum, are to have their energy increased, it must be by impulses timed to their own rate of vibration.

This is the principle of a method of sending many messages over one telegraph-wire at the same instant: A number of tuning-forks or reeds are employed. Electro-magnets are then placed opposite the prongs of the tuning-forks, and the latter pick out of the impulses sent over the telegraph-wire those that correspond to their own rate of vibration. Returning now to the dark lines in the spectrum, we see that an analogy to this phenomenon would be the action of a harp, the strings of which are between us and a certain tuning-fork which is vibrating. If one of the strings is tuned to vibrate to the note of the fork, the note of the fork will be weakened to a very sensitive ear placed near the string, for the energy of the tuning-fork is absorbed in setting this string into vibration. If our means were sensitive enough, we could thus determine by means of a harp the existence of a number of tuning-forks on the side of the harp op-

posite to ourselves, which are too distant to affect our sense of hearing by their own vibrations.

In order to see the bright lines of the elements, it is necessary to employ an observing telescope, for the vapor of sodium, for instance, would not give a sufficiently powerful light to project an image on the screen. The arrangement of the lenses in a spectroscope can be seen in Fig. 149. The beam, passing through the slit *S*, is rendered nearly parallel by the lens, *A*, of long focus. It strikes upon the prism and is refracted and dispersed, and the refracted rays, entering the lens *B*, produce an image of the slit, which in turn is magnified by the combination of lenses in the eye-piece.

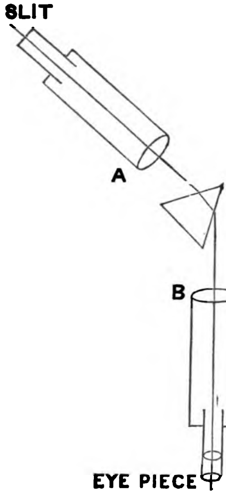

FIG. 149.

EXPERIMENT 198.—Make a small spectroscope by obtaining tin tubes from the tin-man and inserting lenses, which can be obtained from any optician. The interior of the tubes should be blackened to prevent internal reflections. The combination of lenses for an eye-piece can be seen by examining any small telescope. A dark box should be fitted over the prism, with holes for the ends of the two telescopes. The distance of the slit from the object-glass *A* (Fig. 149) must be regulated so that the rays proceeding from the slit, after emerging from the glass *A*, shall be approximately parallel. In order

to accomplish this, focus the telescope, which has the object-glass *B*, upon some distant object. Then remove the prism and place the collimator and telescope in the same line, and move the tube containing the slit *S* until a distinct image is seen in the observing telescope *B;* the rays coming from *S* will then emerge from the object-glass, *A*, parallel.

EXPERIMENT 199.—Having reflected the sunlight upon the slit so that the ray is horizontal and will pass through the axes of the two telescopes, place the prism in the position of minimum deviation, and focus the telescope upon the dark solar lines in the yellow. Then cut off the sun, and place a wire, dipped in salt, in a flame before the slit. A yellow line, called the sodium line, will appear exactly in the place of the darkest solar line in the yellow (the observing telescope should be provided with cross-hairs; see Appendix). It is best to use a platinum wire, one end of which is bent into a loop, which is dipped into fine salt. The other end of the platinum wire can be fused into a glass tube which will serve as a holder.

EXPERIMENT 200.—Repeat the above experiment with chloride of lithium and potassium hydrate, and locate the lines with reference to the colors of the solar spectrum.

EXPERIMENT 201.—Take a thin Florence flask; pour into it half a teaspoonful of nitric acid; drop a bit of copper wire into the acid, and close the flask. In a few moments the flask will be filled with the fumes of nitrous acid. Place the flask before the slit of a spectroscope and examine the sunlight which has passed through the vapor. The solar spectrum will be found to be crossed by a great

number of vertical bands which constitute the absorption spectrum of nitrous acid.

EXPERIMENT 202.—Make a vessel with parallel glass sides, the sides being eight centimetres apart. Fill the cell with a solution of sulphate of copper and place it in front of the slit. The solar spectrum will be absorbed except in the green and blue. Place a second cell in front of the one at the slit, so that the sun's rays will pass through both cells. It will be found that the light which has already passed through the solution of copper, and has lost its red and yellow rays, passes without further absorption through another layer of sulphate of copper.

EXPERIMENT 203.—Draw diagrams of the portions of the solar spectrum absorbed by a solution of indigo.

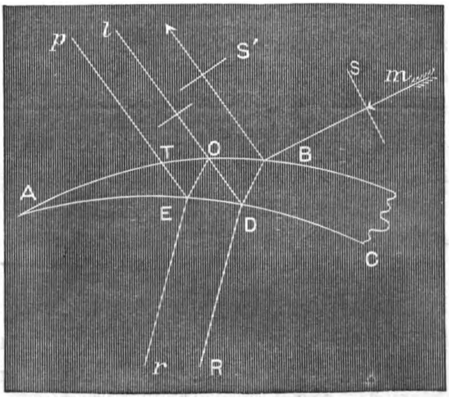

FIG. 150.

Let *A B C D* (Fig. 150) be a section of a thin film like that of a soap-bubble. *m B*, a ray of light striking the surface of the bubble at *B*, will be partly refracted in the direction *B D*, and will emerge in the direction *D R*. It will also suffer internal reflection at *D*, *O*, and *E*, giving rise to rays *O l*, *T p*, and to *E r*. The luminiferous surface

S, perpendicular to the incident ray *m B*, will correspond to a luminiferous surface at *S'* after reflection. The corresponding luminiferous surface perpendicular to the ray *O l* will be behind that at *S'* by the distance *B D* plus *D O*, and therefore interference can take place. The following experiment will show the brilliant colors due to interference:

EXPERIMENT 204.—Into a pint bottle, half filled with distilled water, put one ounce of shavings of white Castile soap. Shake the bottle well and add water until the soap completely dissolves and the solution is clear. Then add a gill of glycerine and strain the mixture. If liquid glycerine soap can be obtained, a suitable solution of this in distilled water will answer. Dip the mouth of an ordinary tumbler into this solution, and having placed the tumbler so that the film is vertical and reflects the light from a window, you will observe beautiful bands of brilliant colors crossing the film.

EXPERIMENT 205.—If a beam of light can be obtained in a dark room, receive it upon the film and reflect it upon a white screen. The phenomenon can thus be shown to a large class.

From Fig. 150 we see that the path of the rays reflected from the front surface of the film and the path of those reflected from the back surface differ by twice the thickness of the film of water, and whenever the difference of path brings the crest of one wave directly over the trough of the other, we have complete interference or darkness. This happens whenever the path of the beam in the film is an even number of half wave-lengths.

The dark lines in the solar spectrum are called Fraunhofer's lines, since he first observed them. Their wave-length can be obtained by a diffraction grating. The most prominent lines are given in the following table. Calling the wave-length λ, we have $\lambda = \frac{\text{velocity}}{\text{number of vibrations}}$. The velocity of light is 300,000,000 metres per second.

Portions of the Solar Spectrum.		Length of waves in millimetres.	Number of waves per second.
Dark red	A	.000760	395×10^{12}
Orange	C	.000656	458×10^{12}
Yellow	D	.000589	510×10^{12}
Green	E	.000527	570×10^{12}
Blue	F	.000486	618×10^{12}
	G	.000431	697×10^{12}
Violet	H	.000397	760×10^{12}

Thus it will be seen that the longest wave-lengths of light which belong to the red and ultra-red rays are approximately $\frac{760}{1000000}$ of a millimetre, and 395,000,000,000,000 of these waves strike the eye in one second, while the tuning-fork which gives 256 vibrations per second, and which corresponds to the middle C of the piano-forte, sends forth wave-lengths 1.30 metres in length. The velocity of sound is about 333 metres per second in air.

Our experiments with soap-films show us some of the most striking phenomena of interference of light. The phenomena of diffraction also exhibit the interference of waves.

Experiment 206.—Take a common spectacle-lens, *A* (Fig. 151), of two or three feet focus, and an eye-lens, *B*, of one or two inches focus. Mount them

by surrounding them by a ring of cork and pushing them into pasteboard or tin tubes—the tube containing the eye-piece slipping within the tube containing

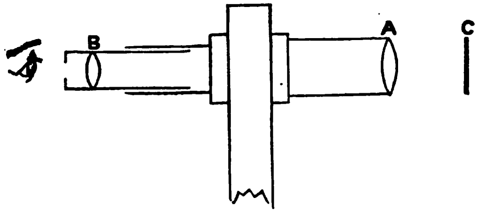

FIG. 151.

the object-glass *A*. The interior of the tubes should be blackened, and the lenses set perpendicular to the axis of the telescope. Mount the telescope thus made upon a suitable stand. An eye-hole should be placed in front of *B*, a little within its focus. Draw the eye-tube out until an image of a distant luminous point is seen distinctly in the telescope. Then place the support of the apparatus at *C*, between the object-glass *A* and the luminous point.

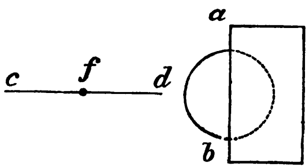

FIG. 152.

The light should be at a considerable distance. For a distance from twelve to eighteen feet the object-glass should have a focal length of one to one and a half feet. As a luminous point the sun's rays striking a convex mirror can be used. Cover one half the object-glass by a sheet of tin-foil (Fig. 152), running the straight thin edge of the foil across the middle, *a b*,

of the glass. The image of the luminous point f is now spread out by diffraction into the line $c d$. The luminiferous surface is unaltered in the direction parallel to the edge $a b$, and the lens brings these rays to a focus at f, but the surface will be formed of arcs nearly circular in a direction perpendicular to the edge $a b$, and the diffracted rays originating at this edge will not be brought to a focus by the object-glass, and hence are opened out into a line of light.

Make a narrow slit about one fiftieth of an inch broad, and place it between the object-glass A and the distant source of light. The central image will

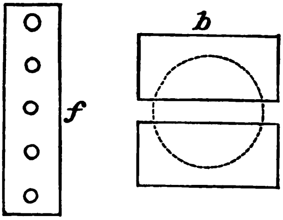

FIG. 153.

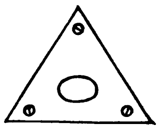

FIG. 154.

be elongated, and colored spectra will be seen on both sides of f (Fig. 153).

EXPERIMENT 207.—Study the effects produced by giving the aperture different forms, such as a triangle, a square aperture, a number of small circular holes punched in a thin metallic plate; also use a fine wire gauze.

EXPERIMENT 208.—Place a spectacle-lens of long focus on a flat piece of glass, having carefully cleaned them both (Fig. 154). Place this combination between two pieces of wood, and by means of screws at three points press the lens and glass together,

and rings of color will be observed around the center of the lens: these are called Newton's rings. The explanation is the same as that of the interference produced by soap-films. View these rings by homogeneous light. Homogeneous light can be produced by burning salt in the flame of a Bunsen burner, or of a spirit-lamp. With this light the colors of the rings disappear, and we have merely dark rings.

The experiments which we have now tried upon diffraction and the interference of light prepare us to understand how the energy of the solar spectrum can be measured.

In the case of radiant energy the movements of the ether are perpendicular to the direction of propagation of the wave; that is, a particle of the ether moves like a boat at anchor—up and down—while the wave passes beneath it. This motion is also, it will be observed, a pendulum motion. It is the same as that made by the prong of a tuning-fork observed by an eye placed directly in line with the vibrating prong, and can be represented also by the ordinary pendulum by connecting a string to the pendulum-bob, passing it through an eyelet *E*, in the line of rest of the pendulum, and attaching a weight *w* to the other end; as the pendulum-bob swings to and fro, the weight *w* executes periodic movements similar to a particle of ether when a wave of light passes through it.

In the case of sound-waves, which are propagated through ordinary air and gases and solids, and not through or by the ether, the vibration of the particles is in the direction of the propagation of the wave. Perhaps the simplest illustration of this is

the phonograph, which consists of a circular metallic disk supported around its edges, and provided with a point or stylus at its center. This point rests upon a grooved cylinder covered with tin-foil.* Speak into the mouth-piece surrounding the disk and turn the cylinder: the sound-waves will be seen to make little dots upon the tin-foil. These dots are due to impulses perpendicular to the disk, and therefore in the direction of the sound-waves. On revolving the cylinder in the opposite direction, the little dots or depressions will communicate impulses to the disk, which in turn will give sound-waves. It is evident that sound-waves can be refracted as well as light-waves, and Herschel's illustration of the march of soldiers across a plowed field can also be used in reference to the front of a sound-wave, which impinges upon the line of separation between two media. The striking difference between the waves of sound and those of light is due to the difference in velocity of their propagation, the velocity of sound-waves in air being about 1,000 feet per second, while that of light is 180,000 miles per second, and also the difference in their size; the longest visible wave of light is not more than .000760 of a millimetre, while the longest wave of sound sensible to us as a musical sound is about sixteen feet. Our apparatus for measuring heat- and light-waves must therefore be capable of measuring very much smaller quantities than our sound apparatus.

This difference in dimensions is shown in the phenomenon of diffraction. On looking through a very narrow slit at a flame, we perceive bright and

* The cylinder moves along its axis of revolution by means of a screw, and the stylus therefore draws a helix upon the tin-foil.

dark spaces on each side of the light of the slit. The light rays appear to be bent out of their straight course by the edges of the slit. On sending a sound-wave through a slit, and using suitable apparatus to detect spaces where sound can be heard and where it can not be heard on each side of the slit, we find that with a suitable sized slit the phenomena of diffraction of sound can also be observed. The size of the slit, however, is enormous compared with that which is used to obtain the phenomena of the diffraction of light. In both cases the slit must be of the order of dimension of the waves which are to be affected in their passage. There is very little difficulty in getting the length of the waves of sound, but we naturally infer, from the size of the waves of light, that it is much more difficult to obtain their dimensions. The method of diffraction is employed for this purpose.

Let us return to the analogy we have used in the case of the refraction of light, and suppose that our flock of birds of various colors and with different rate of wing-movements, all advancing with the same velocity, should endeavor to fly through a narrow slit between high rocks. The sportsman on the side toward which the birds are flying would then see a number of birds of all colors flying directly through the middle of the opening and not diverted by the sides of the slit. On each side, however, of this line passing perpendicularly through the middle of the slit, and in the horizontal plane passing through the middle of the slit, there will be spaces where there will be no birds, and where there are birds advancing in an extended front; the red birds being most diverted from the central line, and the

violet birds, having the shortest length of wing, the least diverted. The disappearance of birds from this horizontal plane can be explained by the clashing of wings of two birds of the same length of wing, one bird moving its wings upward while the bird above it moves its wings downward. In this way the work the birds do in striving to maintain their common velocity in passing through the slit, gives a spectrum, so to speak, of birds separated by spaces where there are no birds, on each side of the central line passing perpendicularly through the middle of the slit. Passing now to the case of light-waves, we see that two waves of the same length, counting the length of a wave as the distance from the height of an elevation to the depth of a depression, advancing together, will superimpose themselves on each other, and the resulting height and depression will be due to their combined movement. If, however, the following wave should be one half a wave-length behind, the upward movement of one will be opposed by a downward movement of the other, and there will be no motion, and consequently no light.

If, therefore, we can make our slit so small that the difference in the paths of the deflected rays of light can in certain cases be whole wave-lengths and half-waves, we shall have spectra of light separated by dark spaces, the spectra being produced by the superposition of waves of the same length and of the same path, while the dark spaces are due to the interference of waves of the same length but whose paths differ by one half a wave-length.

The best way to form slits is found to be to rule very fine lines on glass or speculum-metal, the light

in the case of glass passing through it, and in the case of lines ruled on speculum-metal being reflected from the polished surface of the metal; in both cases the inequality in the surfaces affording a slight difference of path for the different waves of light. Let us first take the case of the glass diffraction grating. Let S (Fig. 155) be the distance between any two fine lines. Let the angle a represent the angular interval between the direction of the line passing directly through the slit, and the direction of the ray of any color whose wave-length we wish to measure. From our preceding illustrations it is evident that light of this color is found in one of the diffraction spectra when the difference in position between the wave-front of the light striking at B, and that wave-front of the same colored light arriving at C, after striking at A, is a whole wave-length. If this difference were half a wave-length, the two waves would interfere, and no light would be seen at this angular interval. Hence, $S \sin a = \lambda =$ wave-length. Since $ABC = a$, the sides being mutually perpendicular.

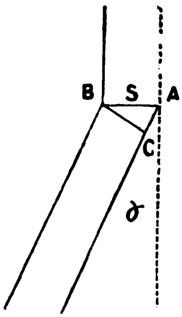

FIG. 155.

It is customary to measure the angular interval between the same points in the two spectra which are formed on both sides of the line perpendicular to the grating which also passes through the slit employed to throw a beam of parallel light upon the grating.

The apparatus for measuring wave-lengths therefore is a spectrometer, in which the prism of Ex-

periment 187 is replaced by a diffraction grating mounted at the middle of a graduated circle; the angular intervals between the line of light passing through the collimator perpendicular to the grating and any line of light or reversed solar line in any spectra being measured on this graduated circle. The spectrum formed by rays the least diverted is called a spectrum of the first order; then follows a dark space, and then a spectrum of the second order, and so on.

These spectra are called normal spectra, since they are not distorted by a medium such as that of a prism.

Modern measurements of wave-lengths are therefore made with a refined spectrometer which is capable of measuring angles with great precision, and a diffraction grating, which is generally ruled on highly polished speculum-metal. The difference in path of the waves in this case is produced by reflection from the grating on the metal. The substance of the metal absorbs a portion of the energy of the light falling upon it, but this portion is extremely small, and is not far from the order of errors which are due to the most refined apparatus we employ in measuring the wave-lengths. The distance *S* between the lines of the grating can be measured under a microscope. It is generally recorded by the dividing engine which rules the grating. On measuring by means of the thermopile the distribution of heat in the spectrum, it is found that the maximum energy occurs in the orange. There is, then, a great field beyond the red invisible to the eye, in which nevertheless the waves of energy can affect the molecules of matter.

In the direction of the ultra-violet there is less energy manifested as heat, and less as light. The very short waves of the ultra-violet, moving with great velocity, are capable, however, of disturbing the equilibrium of the grouping of molecules in many substances, especially in the salts of silver. Thus we have at this end of the spectrum great photographic sensitiveness.

EXPERIMENT 209.—Having formed a solar spectrum, receive it upon a screen *S* (Fig. 156). Place this screen so as to cut off half the visible spectrum, and

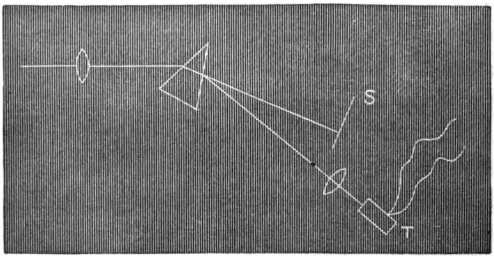

FIG. 156.

allow the rays of the remainder, brought to a focus by a lens, to fall upon a thermopile, *T*, made of a great number of junctions of bismuth and antimony, which is connected with a very sensitive galvanometer, the deflections of which are observed by means of a telescope and scale. It will be found by moving the screen that the two halves of the spectrum thus collected by the lens do not give the same effect; and that a less portion near the red end is equivalent to the larger portion of the remainder of the spectrum. If the instruments are sensitive enough, it will be found that there are dark rays of heat be-

yond the red, which can affect the thermopile, but are not able to affect the eye of the observer. If we should represent by a curve the distribution of heat in the solar spectrum, produced by means of a diffraction grating, it will be found that the maximum heat energy is in the red and orange.

Spectrum analysis is a study of the conservation of energy. The application of a spectrometer to the detection of metals by the existence of their characteristic bright lines, or to the mapping of the dark lines in the solar spectrum, is merely an application of the principle of the conservation of energy.

To show that the light-waves carry also heat, and that the two are merely manifestations of energy, and that we can not separate our consideration of light from that of heat, a study of the distribution of heat in the solar spectrum is necessary. This study requires very sensitive apparatus, for the amount of heat in any one portion of the spectrum is very small. Formerly, this study was conducted entirely by means of the spectra produced by glass or liquid prisms. It was soon discovered, however, that these prisms distorted the spectrum, giving a great distribution of heat near the red end of the spectrum and very little in other parts. Before explaining the more refined methods of subsequent observers, let us explain the old methods. Instead of a thermopile, electrical phenomena can be used in another way to illustrate the distribution of energy. The electrical current heats a wire, and in thus heating a conductor internal work is done upon the wire. In turn, if we apply external heat to the wire, the electrical resistance of the wire is increased.

If we should, therefore, form a Wheatstone's bridge, and make one of the branches very sensitive to the rays of heat, that is, so that a slight change in temperature will produce a great change in electrical resistance, we could use this bridge to measure temperature as well as electrical resistance.

EXPERIMENT 210.—Measure the variations in temperature in the neighborhood of a gas-flame. Make one of the arms, *R* (Fig. 157), of a bridge of fine platinum wire, and cover it with lamp-black, lamp-black being the best absorbent of heat; place a similar coil, *R′*, in the opposite branch, in water, in which is also a thermometer. The connections of *R* and *R′* with the corners of the bridge should be made of thick copper wire, of which the change of resistance can be neglected. As we move the wire *R* toward the flame, its resistance will change, and we must heat the water in which *R′* is immersed in order to restore the balance of the bridge. The increase of temperature is indicated by the thermometer in the water surrounding *R′*. This method can evidently be used to ascertain the temperature of places where a mercury thermometer can not be conveniently used, such as the depth of the sea, or the interior of furnaces. It has certain advantages over a thermo-electric junction, for a thin wire can be moved across the solar spectrum and will indicate

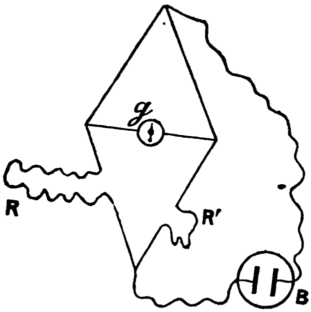

FIG. 157.

by a change in electrical resistance the temperature of different portions.

The extreme sensitiveness of metals to changes of temperature is illustrated by this use of the Wheatstone bridge, in which electricity makes these changes evident. This sensitiveness can also be manifested by what is called the radiophone. A beam of light is interrupted a great number of times a second, and its pulsations are received upon a thin metallic plate which closes one end of an ordinary speaking-tube. On placing the other end of the speaking-tube to the ear, the note produced by the number of interruptions of the beam of light will be heard. In this case the metallic disk changes its form in unison with the increase and diminution of temperature. On replacing this thin metallic disk with various non-metallic substances, the same phenomenon is observed. Different gases, through which such an intermitted beam is made to pass, also give forth the note caused by the wheel. The apparatus necessary for this experiment is very simple. The chief difficulty is in making the interrupting wheel revolve noiselessly at high speed.

These experiments illustrate the conversion of radiant energy, which reaches us in a beam of light, into molecular agitation and sensible heat, and then into sound; the wave-motion of light in these experiments being finally changed into the wave-motion of sound.

In pursuing the study of physics we find that the principle of the conservation of energy is constantly exemplified, and that even in the subjects of light and electricity we are studying motion, and need a sound knowledge of mechanics in order to test molecular theories. The subject of molecular physics is beyond the scope of this treatise.

APPENDIX.

GENERAL DIRECTIONS.

THE laboratory should be provided with certain reference-books. The following will be found extremely useful:

Daniell's "Principles of Physics."
Everett's "Physical Constants."
Pickering's "Physical Manipulations."
Ganot's "Physics."
Kohlrausch's "Physical Measurements."

An elementary physical laboratory requires merely some suitable tables, with plenty of light. At one or two of the tables gas should be introduced. If gas can not be obtained, the various forms of spirit-lamps, such as are used in little portable cooking-arrangements, can take the place of Bunsen gas-burners. A water-tap is also necessary, and a soapstone sink, sufficiently large to contain the batteries employed. It is convenient to have a large closet, or a portion of the laboratory inclosed, which may be darkened, for mirror galvanometers, and for experiments with lenses.

When the teacher has the requisite leisure and ingenuity, much physical apparatus can be constructed with comparatively little money. The student can readily construct his own galvanometer and make resistance-coils with the aid of one standard box of coils. Instead of making electro-magnets, one can generally purchase at a low price suitable ones which have been discarded by manufacturing electricians. Gravity batteries can be readily constructed by the student, and also bichromate of potash batteries. See pages 212–213.

A list of essential apparatus for a class of six boys is appended, with an approximate estimate of its cost:

A balance weighing to five milligrammes, together with the weights	$60 00
Three Bunsen burners, or spirit-lamps	4 00
Three hydrometers	1 50
Six specific-gravity bottles (bottles with ground-glass stoppers, ten cents apiece)	60
Set of cork-borers	2 00
Mercury	10 00
Air-pump (Richards' aspirator can be obtained of Prang & Co., Boston)	2 50
Thermometers and vernier gauges (Alvergniat Frères, Paris, France; Brown & Sharp, Providence, Rhode Island)	20 00
One box of resistance-coils (Charles Williams & Co., electricians, Boston)	50 00
Four galvanometers	10 00
Two electrometers	50 00
Four tuning-forks	8 00
Four lenses	8 00
Four prisms	8 00
Tools, corks, wire, calorimeters, alcohol, rubber tubing, glass, general supplies	50 00
	$284 60

With the above apparatus a class of twelve boys can be well employed, for the time generally devoted to physics in the high schools, working in sets of three. It is best to gradually duplicate the balances and sets of resistance-coils. The student should make his own sets of small weights and set of small resistance-coils.

USEFUL APPLIANCES.

Cement.—Take of gutta-percha (sheets cut into shreds) one part, and of pine (not coal-tar) pitch one part.

Melt at 100° C. for some time, stirring occasionally till homogeneous. This cement is useful as a substitute for sealing-wax in fitting up apparatus, and fuses at so low a temperature that tight joints may be made even in the presence of water, and it improves every time it is heated.

When a large hole is to be made with a "cork-borer" in a small cork, the danger of splitting can be avoided by wrapping the cork tightly with twine.

Paraffine is an excellent lubricator when thermometers, etc., are to be thrust through tightly-fitting holes in corks.

The interior of a wet bottle can be dried by passing a blast of air from a bellows through a piece of glass tubing which projects into the bottle. It can be more quickly dried by heating the air as it passes through the glass tube by placing a Bunsen burner under the glass tube and heating it to a low red heat.

A simple method of calibrating a thermometer-tube is to separate a thread of mercury (by shaking the thermometer) about 50° long. Run it to the upper end of the thermometer-stem, placing one end at the 100° mark. Then read the other end. Then run it down the stem until one end rests at the 50° mark, and read the other end, and estimate how much too long or too short the divisions are between 50° and 100° compared with those between 0° and 50°. Then separate a thread 25° long, and repeat the operation, using the 25° and the 50° marks in the same way that the 50° and 100° marks were used, and distribute the errors of graduation by comparing all the graduations with the graduations from 0° to 25° as standards, and so on.

Glass-working requires much practice in order to do well. The following hints may be useful:

To smooth the Ends of Tubes.—Warm the end in a flame, and then hold it obliquely in the flame, turning it till the edges are rounded.

To bend Tubes.—Gradually warm the tube in the flame; then hold it a little obliquely, turning it between the thumbs and the fore-fingers until it is yellow. Then remove it from the flame and bend it with one motion.

To close the End of a Tube.—Soften it at the end and pull the end out with another piece of glass. Keep removing the small tail that is left till the latter is very slender; then heat the latter, turning the tube in the fingers.

To fuse Platinum into a Glass Tube.—Heat the glass tube thoroughly at the portion where the platinum wire is to be in-

serted. Then thrust the wire into the glass and pull out the latter into a fine tube. Cut this tube off near the main tube and round the edges. Then insert the red-hot platinum wire and turn it around until perfect contact is obtained.

To join Tubes at Right Angles.—Close one end of each tube, and then direct a pointed flame from a blow-pipe upon the side of one of the tubes. Blow into the tube and gradually produce a swelling on the side of the tube. Finally, by a quick puff, force this swelling open. Round the edges of this opening in the flame. Heat the edges of the other tube and then fuse it to the opening.

To blow a Bulb.—First close the end of a tube by turning it for some time in a flame. After the end has become very hot, blow gently into the other end.

Holes can be bored in glass readily by employing a very small rat-tail file, the end of which has been broken off with a pair of pincers. This broken end is pressed against the glass and kept moistened with a paste made of spirits of turpentine and gum-camphor. The camphor acts in a mechanical way, holding the grinding particles in the cavity made by the end of the file. The end of the file should be broken off slightly from time to time with the pincers.

Glass bottles can be cut in two by running a little pointed flame from a crack started by a file-scratch in the surface of the glass. In order to produce the pointed flame, pull out a small tube of glass to a fine point and break off the end of this point and insert the wide end of the tube in a piece of rubber tubing connected with a gas-cock. The size of the flame can be regulated by the fineness of the point of the glass. The flame should be very small and very pointed.

A soldering-iron, with a comparatively fine point, can be used instead of the small gas-flame.

The edges of a glass vessel can be ground smooth by rubbing the edges on emery-paper moistened with water, or by rubbing the edges on a flat surface of iron covered with emery-powder suitably moistened.

Experiments involving the use of mercury should be performed upon a table, the top of which is provided with ledges. The table should be slightly inclined, in order that the stray mercury may collect at one side.

If millimetre scales can not be obtained, the box-wood scales employed by engineers, which are divided into tenths and twentieths of an inch, can be used.

Accurate vernier gauges or micrometer gauges can be obtained of Alvergniat Frères, of Paris, or of Brown & Sharp, Providence, Rhode Island.

A simple dividing engine is described in Pickering's "Physical Manipulation," vol. i, page 59, which can be constructed at slight expense, and will enable one to graduate paper scales, which are of great service in elementary physical laboratories.

Insertion of Cross-Hairs.—Cut from card-board a little ring with projecting flanges. Stretch two fine fibers of silk—the finest fibers of a silk thread—at right angles to each other. Bend the flanges so that the ring can be slipped along the tube of a telescope and be placed at the focus of the eye-piece.

The telescope and micrometer should be made by the student. The apparatus needed consists merely of tin or brass tubes, and corks to fit the lenses to the tubes or the tubes to each other.

A large tuning-fork, suitable for many of the experiments on motion, can be made by any blacksmith. The length of its prongs should be about fifteen inches, the distance of these apart about four inches, and the section half an inch by a quarter of an inch. It is well that the handle should be long and heavy, in order to afford a counterpoise to the prongs of the fork. A smaller fork made in the same way, of about one half the size of the larger fork, will also be useful. The price of such tuning-forks need not exceed one dollar and a half apiece. Small vises suitable for clamping such forks can be obtained at hardware-stores. These vises can be screwed to loaded boxes or to any suitable supports.

Plane or convex mirrors suitable for galvanometers can be obtained by mail from Elliott Brothers, 449 Strand, London, W. C. The average price of these mirrors is about fifty cents.

If a telescope is used, a plane mirror should be affixed to the magnet; if a spot of light, a convex mirror. A lens of long focus, such as an ordinary spectacle-lens, placed directly in front of a plane mirror, will enable one to use a spot of light. It is there-

fore better to order plane mirrors, for they can be used as concave mirrors by employing a lens placed in front of them, or they can be used with a telescope and scale. The lens, moreover, enables one to vary the distance of the spot of light on a screen, and thus increase or diminish the index beam of light.

Sets of resistance-coils suitable for the experiments in this book can be obtained of Charles Williams & Co., electricians, Boston, Mass. The price is about fifty dollars. With one box of coils other sets of resistances can be made by the students themselves. For many experiments the estimation of resistances by means of Table VIII is sufficient. The correct value of such estimated resistances can be obtained subsequently.

Simple break and make keys can be made by screwing a piece of spring-brass to a board, connecting a wire to the screw, and another wire to a screw upon which the brass can be pressed. There are many forms of reversing-keys. A simple one can be made as follows: Screw four square pieces of brass at the four angles of a square piece of board, having first boiled the wood in oil or covered it with paraffine. The pieces of brass should be about one inch square and half an inch apart Solder wires to each of these brass plates, or screw binding-screws upon them. Connect A and D with the galvanometer or electrometer, and C and B with the battery. If, then, we connect C and A, and B and D, the current may flow from C to A, then to the galvanometer, and may return to B and D, and then to the battery. If, on the other hand, we connect B and A, and D and C, the current passes through the galvanometer in the reverse direction. A suitable bar, provided at its ends with pieces of spring-brass insulated from each other, and capable of revolving about E, can serve to perform the operation indicated.

$A \quad B$
E
$C \quad D$

It is a useless expense for the purposes of elementary laboratory instruction to buy an expensive galvanometer. The student should construct his own galvanometers. With very little care he can make instruments which are as exact and as sensitive as those made by instrument-makers. In making astatic galvanometers, the principal difficulty consists in making the two magnets, which are placed one above the other with poles reversed, exactly equal. A little care will accomplish this. The astatic combina-

tion should then have a very slow time of vibration. It is necessary to employ a magnet outside the galvanometer to bring the astatic combination back to a zero. The astatic galvanometer described in the chapter on thermo-electricity need not cost more than three dollars, and is extremely sensitive to thermo-electric currents.

TABLES.

The following tables embrace only the data necessary for the performance of the experiments in this treatise, and are taken mainly from Kohlrausch's "Physical Measurements." The student is referred to Everett's "Physical Constants" for more extended tables.

TABLE I.

To convert millimetres into inches, multiply by .03937.

To convert metres into inches, multiply by 39.37.

To convert grammes into grains, multiply by 15.44.

To convert kilogrammes into pounds, multiply by 2.205.

1 United States liquid quart = 0.946 litre, a little less than 1 litre.

1 United States dry quart = 1.101 litre, a little more than 1 litre.

1 United States gallon = 3.785 litres, or about $3\frac{8}{10}$ litres.

1 avoirdupois ounce = 0.02835 kilo, or rather less than 30 grammes.

1 Troy and apothecaries' ounce = 0.03110 kilo, or rather more than 30 grammes.

1 avoirdupois pound = 0.45359 kilo, or about $\frac{5}{11}$ kilo.

TABLE II.

Table of Specific Gravities.

Solids.

Standard for solids and liquids is distilled water at 0° C.

Antimony	6.712	Glass, flint	3.400
Beech	0.852	Gold	19.360
Bismuth	9.822	Human body	0.890
Brass	8.380	Ice	0.920
Cork	0.240	Iridium	23.000
Diamond	3.530	Iron, cast	7.210
Ebony	1.187	Iron, bar	7.780

Lead, cast	11.350	Saltpeter	1.900
Oak	0.845	Silver, cast	10.470
Pine	0.650	Sulphur, natural	2.033
Platinum	22.069	Tin, cast	7.290
Quartz	2.650	Zinc, cast	6.860
Rock-salt	2.257		

Liquids.

Alcohol, absolute	0.800	Nitric acid	1.420
Bisulphide of carbon	1.293	Oil of turpentine	0.870
Ether	0.723	Olive-oil	0.915
Hydrochloric acid	1.240	Sea-water	1.026
Mercury	13.598	Sulphuric acid	1.841
Milk	1.032	Water, 4° C., distilled	1.000
Naphtha	0.847	Water, 0° C., distilled	0.999

Gases.

Standard: air at 0° C.; barometer, 76cm.

Air	1.0000	Hydrogen	0.0693
Ammonia	0.5367	Nitrogen	0.9714
Carbonic acid	1.5290	Oxygen	1.1057
Chlorine	3.4400	Sulphuretted hydrogen	1.1912
Hydrochloric acid	1.2540	Sulphurous acid	2.2474

TABLE III.

Coefficients of Expansion for 1° C.

The length L of a body is increased by βL for each degree of increased temperature, and its volume V by $3\beta V$.

	β		β
Lead	0.0000285	Brass	0.000019
Iron	0.000012	Platinum	0.000009
Glass	0.0000085	Silver	0.000019
Gold	0.000015	Zinc	0.000029
Copper	0.0000175	Tin	0.000022

The volume V of quicksilver increases $0.0001815\,V$ for 1°.

TABLE IV.

Moments of Inertia.

Thin bar of length l and weight m. The thickness uniform and very small compared with l. Referred to an axis at right angles to the length l, and passing through its center:

$$K = \frac{m\,l^2}{12}.$$

Right-angled parallelopipedon—a and b two adjacent edges. Axis passing through the center of gravity and parallel to the third edge : $K = m\left(\frac{a^2 + b^2}{12}\right)$.

Cylinder of radius r and length l, referred to the axis of the cylinder : $K = \frac{mr^2}{2}$.

Cylinder of radius r and length l, referred to the axis perpendicular to the middle of the cylinder : $K = m\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$.

Hollow cylinder, r = inner radius, and r_i = outer radius, referred to axis of cylinder : $K = m\left(\frac{r^2 + r_i^2}{2}\right)$.

Sphere of radius r, referred to a diameter : $K = \frac{2}{5}\,mr^2$.

Experimental Determination of Moment of Inertia.

The time of oscillation of the body is observed about the axis, and then the moment of inertia increased by a known amount without changing the field of force, or the directive force, and the time of oscillation again observed.

If t = time of oscillation of the body alone,

t_i = time of oscillation of the body with added weight,

k = moment of inertia :

We have $t_i^2 : t^2 = (K + k) : K$,

or
$$K = k\frac{t^2}{t_i^2 - t^2}.$$

The oscillations must be through very small arcs.

TABLE V.

SPECIFIC HEATS.

Lead	0.0314	Silver	0.0570
Iron	0.114	Zinc	0.0955
Glass	0.19	Tin	0.0562
Gold	0.0324	Ether at 17°	0.516
Copper	0.0951	Alcohol at 17°	0.615
Brass	0.094	Quicksilver	0.0333
Platinum	0.0324	Oil of turpentine at 17°	0.426

Water at 0°	1.0000	Water at 30°	1.0020
Water at 10°	1.0005	Water mean between 0° and 100°	1.0050
Water at 20°	1.0012		

TABLE VI.

Specific Resistances of Metals compared with that of a Column of Mercury at 0°.

Antimony (pressed)	0.360	Lead (pressed)	0.199
Bismuth (pressed)	0.133	Mercury	1.0000
Brass (hard)	0.051	Platinum (soft)	0.0918
Copper (hard)	0.0166	Silver (soft)	0.0153
Copper (soft)	0.0162	Silver (hard)	0.0166
German-silver (hard)	0.212	Tin	0.134
Gold	0.0209	Zinc	0.0571
Iron (soft)	0.0986		

If we call the specific resistance of a metal s, the resistance R of a length l of section q expressed in square millimetres will be

$$R = \frac{sl}{q}.$$

The numbers given in the table are only approximately correct for the commercial metals.

TABLE VII.

Comparison of Measures of Electric Current-Strength.

A current-strength which is measured in—	Must be multiplied by the following numbers to reduce it to—				
	Cubic cm. water-gases per min.	Mgr. water per minute.	Mgr. copper per minute.	Mgr. silver per minute.	Magnetic measure $\frac{\text{Mm.}^{\frac{1}{2}}\ \text{mgr.}^{\frac{1}{2}}}{\text{Sec.}^2}$
Cubic cm. water-gases per min.	....	0.5363	1.889	6.432	0.9579
Mgr. water per min.	1.865	....	3.522	11.99	1.786
Mgr. copper	0.5294	0.2839	....	3.405	0.5071
Mgr. silver	0.1555	0.0834	0.2937	....	0.1489
Magnetic meas. $\frac{\text{Mm.}^{\frac{1}{2}}\ \text{mgr.}^{\frac{1}{2}}}{\text{Sec.}^2}$	1.044	0.5599	1.972	6.714	

TABLE VIII.

NEW STANDARD WIRE GAUGE.

Temperature 15° C. Hard-drawn pure copper wire.

No.	Diameter in inches.	Resistance per yard in ohms.	No.	Diameter in inches.	Resistance per yard in ohms.
0	.324	.000299	22	.028	.0400
2	.276	.000412	24	.022	.0649
4	.232	.000583	26	.018	.0969
6	.192	.000851	28	.0148	.143
8	.160	.00123	30	.0124	.204
10	.128	.00192	32	.0108	.269
12	.104	.00290	34	.0092	.371
14	.080	.00490	36	.0076	.544
16	.064	.00760	38	.0060	.872
18	.048	.0136	40	.0048	1.33
20	.036	.0242	42	.0040	1.96

TABLE IX.

ELECTRO-MOTIVE FORCE OF CONSTANT BATTERIES (J. C. Maxwell).

				Concentrated solution of	Volt.
Daniell I.	Amalgamated zinc.		$H_2SO_4 + 4$ Aq.	$CuSO_4$ Copper.	1.079
" II.	"	"	$H_2SO_4 + 12$ Aq.	$CuSO_4$ "	0.978
" III.	"	"	$H_2SO_4 + 12$ Aq.	$CuNO_3$ "	1.00
Bunsen I.	"	"	$H_2SO_4 + 12$ Aq.	HNO_3 Carbon.	1.964
" II.	"	"	$H_2SO_4 + 12$ Aq. sp. gr. 1.38	"	1.888
Grove.	"	"	$H_2SO_4 + 4$ Aq.	HNO_3 Platinum.	1.956

TABLE X.

MEAN INDICES OF REFRACTION AND DISPERSIONS OF SEVERAL BODIES.

	Index of refraction.	Dispersion.
Crown glass (mean)	1.53	0.022
Flint glass (mean)	1.60	0.042
Water	1.336	0.0132
Alcohol	1.372	0.0133
Carbon disulphide	1.68	0.0837
Canada balsam	1.54	
Air	1.000294	

TABLE XI.

NUMBERS FREQUENTLY USED.

Coefficient of expansion of gases, 0.003665 ($\frac{1}{273}$).
Latent heat of water = 79.4.
Latent heat of steam at 100° C. = 540.
Mechanical equivalent of heat:
 1 pound water heated 1° F. = 772 foot-pounds.
 1 pound water heated 1° C. = 1390 foot-pounds.
 1 gramme water heated 1° C. = 424 gramme-metres.
$\pi = 3.1415926$.
Square root of 2—1.4142.
Square root of 3—1.7320.
Force of gravity at equator in centimetres, 978.0.
Force of gravity at pole in centimetres, 983.2.
Cubic inch in centimetres, 16.387.
Weight of 1 cubic inch of water in grains at 62° F. = 252.458
Foot-pound in kilogramme-metres, .13825.
Dyne in grammes, approximate, .00102.
Erg in gramme-centimetres, approximate, .00102.
Velocity of sound in air, in centimetres at 0° C. = 33030.
Latent heat of fusion of ice, 79.4.

TABLE XII.

Trigonometrical Functions.

Angle.	Sine.		Tangent.		Cotangent.		Cosine.		
0°	0.000	17	0.000	17	∞		1.000	0	90°
1	0.017	18	0.017	18	57.29		1.000	0	89
2	0.035	17	0.035	17	28.64		0.999	0	88
3	0.052	18	0.052	18	19.08		0.999	1	87
4	0.070	17	0.070	17	14.30		0.998	2	86
5	0.087	18	0.087	18	11.43		0.996	1	85
6	0.105	17	0.105	18	9.514		0.995	2	84
7	0.122	17	0.123	18	8.144		0.993	3	83
8	0.139	17	0.141	17	7.115	811	0.990	2	82
9	0.156	18	0.158	18	6.314	643	0.988	3	81
10°	0.174	17	0.176	18	5.671	526	0.985	3	80°
11	0.191	17	0.194	19	5.145	440	0.982	4	79
12	0.208	17	0.213	18	4.705	374	0.978	4	78
13	0.225	17	0.231	18	4.331	320	0.974	4	77
14	0.242	17	0.249	16	4.011	279	0.970	4	76
15	0.259	17	0.268	19	3.732	245	0.966	5	75
16	0.276	16	0.287	19	3.487	216	0.961	5	74
17	0.292	17	0.306	19	3.271	193	0.956	5	73
18	0.309	17	0.325	19	3.078	174	0.951	5	72
19	0.326	16	0.344	20	2.904	157	0.946	6	71
20°	0.342	16	0.364	20	2.747	142	0.940	6	70°
21	0.358	17	0.384	20	2.605	130	0.934	7	69
22	0.375	16	0.404	20	2.475	119	0.927	6	68
23	0.391	16	0.424	21	2.356	110	0.921	7	67
24	0.407	16	0.445	21	2.246	101	0.914	8	66
25	0.423	15	0.466	22	2.145	95	0.906	7	65
26	0.438	16	0.488	22	2.050	87	0.899	8	64
27	0.454	15	0.510	22	1.963	82	0.891	8	63
28	0.469	16	0.532	22	1.881	77	0.883	8	62
29	0.485	15	0.554	23	1.804	73	0.875	9	61
30°	0.500	15	0.577	24	1.732	67	0.866	9	60°
31	0.515	15	0.601	24	1.664	64	0.857	9	59
32	0.530	15	0.625	24	1.600	60	0.848	9	58
33	0.545	14	0.649	26	1.540	57	0.839	10	57
34	0.559	15	0.675	25	1.483	55	0.829	10	56
35	0.574	14	0.700	27	1.428	52	0.819	10	55
36	0.588	14	0.727	27	1.376	49	0.809	10	54
37	0.602	14	0.754	27	1.327	47	0.799	11	53
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41	0.656	13	0.869	31	1.150	39	0.755	12	49
42	0.669	13	0.900	33	1.111	39	0.743	12	48
43	0.682	13	0.933	32	1.072	36	0.731	12	47
44	0.695	12	0.966	35	1.036	36	0.719	12	46
45°	0.707		0.000		1.000		0.707		45°
	Cosine.		Cotangent.		Tangent.		Sine.		Angle.

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